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NAVAL CIVIL ENGINEERING LABORATORY
Port Hueneme, California

Sponsored by
NAVAL FACILITIES ENGINEERING COMMAND
Alexandria, Virginia

USER'S MANUAL FOR MODCAL — BOUNDING SURFACE SOIL PLASTICITY
MODEL CALIBRATION AND PREDICTION CODE (VOLUME II)

February 1983

An Investigation Conducted by
DEPARTMENT OF CIVIL ENGINEERING
University of California
Davis, California

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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in²	square inches	6.5	square centimeters	cm²
ft²	square feet	0.09	square meters	m²
yd²	square yards	0.8	square meters	m²
mi²	square miles	2.6	square kilometers	km²
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft³	cubic feet	0.03	cubic meters	m³
yd³	cubic yards	0.76	cubic meters	m³
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm²	square centimeters	0.16	square inches	in²
m²	square meters	1.2	square yards	yd²
km²	square kilometers	0.4	square miles	mi²
ha	hectares (10,000 m²)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m³	cubic meters	35	cubic feet	ft³
m³	cubic meters	1.3	cubic yards	yd³
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F
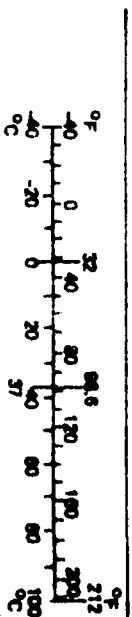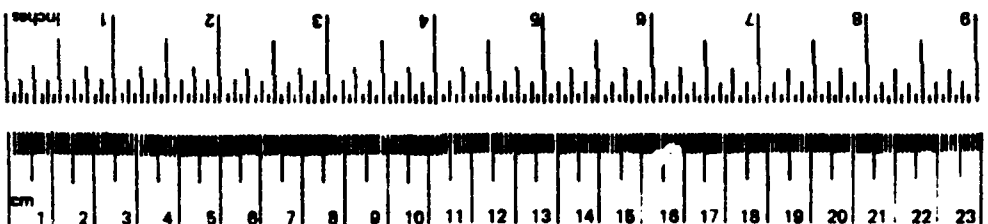
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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*
In recent years, much research has been directed towards the development of sophisticated constitutive models which can more realistically account for the diverse stress-strain phenomena exhibited by soils and other earth materials. Most of these models employ parameters whose optimal values can only be established through a trial and error curve fitting process,

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with the objective being to obtain the best overall fit to a given experimental relation or set of observed responses. As a result, the accuracy and efficiency of this model calibration process can be highly dependent on both the subjectivity of the user as well as his familiarity and expertise with the particular constitutive formulation. In order to minimize this user dependence, and thereby significantly reduce the complexity of the model calibration process, a computer-aided automated procedure has been developed and tested. The computer code employs a Quasi-Newton optimization strategy to locate that set of parameter values which minimizes the discrepancy between the model predictions and the experimental observations included in the calibration data base. Through application to a number of real soils, the automated procedure has been found to be an efficient, reliable and economical means of accomplishing model calibration. Although the code was developed specifically for use with the Bounding Surface plasticity model, it can readily be adapted to other constitutive formulations. Since the code greatly reduces the dependence of calibration success on user expertise, it significantly increases the accessibility and usefulness of sophisticated material models to the general engineering community.
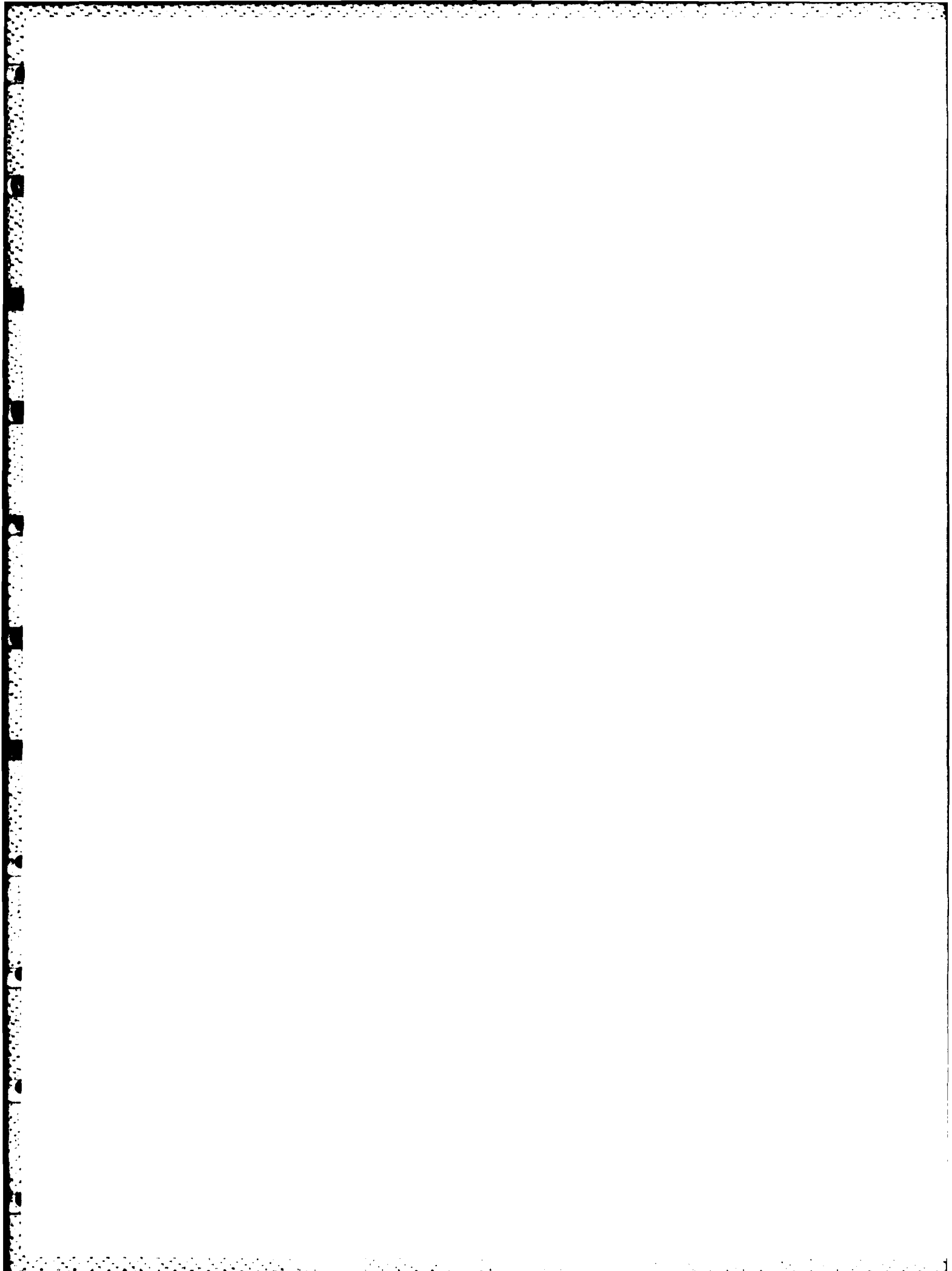
TABLE OF CONTENTS

I.   STRUCTURE AND CAPABILITIES

1.1)   Introduction

Much research has been directed in recent years towards the development of sophisticated constitutive models which can more accurately account for the diverse stress-strain phenomena exhibited by soils and other earth materials. One of the most promising of these new models is a plasticity based formulation known as the Bounding Surface model. The Bounding Surface formulation was originally introduced for metals by Dafalias and Popov (1974, 1975) and Dafalias (1975), and later extended to cohesive soils by Dafalias (1979) and Dafalias and Herrmann (1980, 1982). The model is built within the framework of traditional critical state soil mechanics and employs the concept of a bounding surface in stress space. It has been shown to have the ability to accurately capture both the drained and undrained behavior of clay type soils, at any overconsolidation ratio, under either monotonic or cyclic loading.

In its most general form, the Bounding Surface model requires the determination of 19 separate constitutive parameters, including 2 initial state properties, 5 traditional material constants, whose values may be directly obtained from simple well known laboratory experiments, and 12 model constants, which must be indirectly established through a trial and error curve fitting process using the results of undrained triaxial testing. A general summary of the various properties is presented by Herrmann et al (1980), and a more detailed description of both the qualitative and quantitative influence of each parameter is provided by DeNatale (1982).

This breakdown of model constants is common to most, if not all, of the soil formulations introduced in recent years. Determination of the directly measureable or "fixed" parameters is straightforward and readily accomplished. Determination of the remaining "free" parameters, however, can make the

2

calibration procedure prohibitively difficult. Rather than being measured directly from a particular portion of a specific laboratory test, these so-called "free" parameters must be established by trial and error, with the objective being to obtain the best overall fit to a given experimental relation or set of observed responses. As a result, the accuracy and efficiency of the calibration process can be strongly dependent on both the subjectivity of the user as well as his expertise with the particular material model.

In formulations such as the Bounding Surface model, which employ a small number of material parameters whose roles in the constitutive formulation are each well defined, the calibration process becomes systematic and straightforward. However, reliance on user expertise is still high, since all manual curve fitting procedures, by their very nature, require both judgement (in deciding just what constitutes the "best" overall fit) and familiarity (in deciding how much each parameter's value must be changed to improve a given prediction).

In order to simplify the model calibration process, a computer code has recently been developed by DeNatale (1982) and tested on a variety of real soils. The code employs a Quasi-Newton optimization strategy to locate that set of parameter values which minimizes the discrepancy between the model predictions and the experimental observations included in the calibration data base. Because this new computer aided procedure greatly reduces the dependence of calibration success on user expertise, it significantly increases the accessibility and usefulness of sophisticated material models to the general engineering community.

1.2) The Objective Function

Since the calibration of a constitutive model involves minimizing the error, or residual, between the observed and predicted material response, the

3

process can quite naturally be viewed as an optimization problem. Hence, in order to develop a computer directed calibration procedure, it is necessary to (i) construct an objective function to serve as a scalar measure of the goodness of a particular solution, and (ii) select a search strategy to enable the minimum of this function to be located in an efficient and reliable manner. In both the computer code and the discussion to follow, it is assumed that the model predictions are closely enough spaced so that, by joining adjacent points with linear segments, it is possible to obtain a good approximation to a smooth curve (and thus the concept of a prediction "curve" is valid).

In forming an objective function, the total error between the experimental observations and model predictions could be expressed by either (i) summing the discrete residuals at each of the experimental points included in the calibration data base, or (ii) for each response relation, fitting smooth polynomials through both the observed and predicted data, and then integrating numerically to obtain the area (or residual) between the two curves. In the present study the "discrete" approach was employed, since it appeared to be the most efficient and versatile of the two.

In most data fitting and regression routines, the problem is generally posed in terms of one or more independent variables x and a dependent variable $y=y(x)$. The best fit to the data is then obtained by minimizing some scalar measure which reflects the error in the dependent variable y. This definition of error will hereafter be referred to as the "vertical" measure.

In geotechnical engineering, a number of different variables are typically used to define the soil response. In a conventional undrained triaxial compression test, for example, the response might be characterized in terms of such quantities as the mean normal effective stress p', the deviator stress q, the pore water stress u, and the axial strain ϵ_1. The distinction between independent and

4

dependent parameters is not at all clear. In a so-called "strain-controlled" test, it could perhaps be argued that ϵ_1 represents the independent variable; therefore a vertical measure may be appropriate when considering such relations as q vs ϵ_1 and u vs ϵ_1. But if the data were portrayed as a stress path in q-p´ space, both axes would then represent dependent parameters, and the traditional classification would again break down.

For those cases in which all aspects of the soil response can be expressed in terms of a single quantity (by using, for example, p´ vs ϵ_1, q vs ϵ_1 and u vs ϵ_1 rather than q vs p´, q vs ϵ_1 and u vs ϵ_1), a vertical measure may be reasonable. However, if the distinction between independent and dependent variables cannot be made clear, there is no more reason to use a vertical measure (y=y(x)) than there is to use a horizontal measure (x=x(y)). In these cases it is probably more appropriate to use a measure such as the shortest distance between the experimental observation and prediction curve. While there is no reason to suspect that this alternative is theoretically more sound, such a "Euclidean" measure is probably closer to what one intuitively uses when estimating, by sight, the error between two curves.

In addition to defining a direction along which the error will be measured, it is also necessary to consider whether the sum of the signed, absolute or squared residuals should be minimized. In regression analysis, a squared measure has traditionally been used. The squared measure not only yields a unique solution, but also has additional statistical relevance (as described, for example, by Alder and Roessler (1968)). The absolute measure likewise yields a unique solution, but is generally avoided due to the mathematical difficulties customarily associated with the occurance of absolute values. The signed measure is only rarely used, because it can lead to non-unique solutions.

Since there is no theoretical reason to select one or the other, the calibration code permits ei´ her absolut´ or squared residuals to be used to form

5

the objective function. When these two options are combined with the choice of either vertical or Euclidean measure (as described above), the user then has four possible ways to define the error at a point. Recent research by DeNatale (1982) has shown that the location of the global minimum remains essentially the same, regardless of which options are used. However, preliminary applications to a variety of artificial and real soils indicate that the absolute-Euclidean measure results in a more well-behaved objective function that can be most easily minimized.

In order to combine the residuals at various points within a given relation (for example, the q vs ϵ_1 relation from a test at OCR=1), or from relations of the same kind obtained from different tests (for example, the u vs ϵ_1 relation from tests at OCR=1 and 2), it is necessary to first define what is meant by "equal" error. A given solution could be defined to be equally good at two points A and B if either (i) there was the same absolute error at both A and B, or (ii) there was the same relative error at A as at B. The decision as to which definition of equality should be used is entirely up to the user. The computer code permits the use of either extreme, or any point in between. Consequences of the various choices are discussed further by DeNatale (1982). It should be noted that most physical analogies lie somewhere in the middle —a given dial gauge or pore pressure transducer may be accurate to within α%, but may fail to register any meaningful readings below a magnitude of β.

The code permits any number of tests, relations and/or individual observations to be included in the calibration data base. Because the various response relations will generally be of different dimensions (such as stress vs stress, stress vs strain, strain vs strain, etc.), all data is nondimensionalized so that errors from different relations can be properly combined. Different weights may be assigned to specific components of the data base if it is felt that certain

6

tests, relations or observations are more reliable or representative than others, or if it is necessary to have the final model predictions fit some data more closely than others. The consequences and proper role of weighting factors is again discussed by DeNatale (1982).

1.3) The Optimization Strategy

An extremely large number of optimization strategies have been suggested over the last 30 years, with the performance of a given approach being strongly dependent on the particular type of problem to which it is applied. Hence, it is generally agreed that there is no single best algorithm, but, rather, only strategies which perform best when applied to certain classes of problems.

In selecting the most suitable approach, a key factor is whether or not first and second derivative information can be readily obtained. With the Bounding Surface model, as with most sophisticated material models, the governing equations are so complex that it is essentially impossible to directly relate the relevant response parameters (such as p', q, u, ϵ_1, etc.) to the constitutive parameters employed by the formulation. Thus, the objective function must be formed by summing a series of discrete weighted residuals, and therefore first and second derivative information is not explicitly available.

Under these conditions, the current consensus among those most active in optimization research (including, for example, Fletcher (1980) and Gill et al (1981)) is that a Quasi-Newton strategy with finite difference approximations to derivatives will, if properly implemented, generally exhibit the most efficient and reliable performance. Hence, a Quasi-Newton strategy was incorporated into the calibration code to direct the search.

The Quasi-Newton algorithms avoid most of the difficulties associated with Newton's method (as outlined, for example, by Murray (1972)), while retaining the roubstness of the second derivative methods. In contrast to Newton's method,

7

where the Hessian G is evaluated directly at each iteration, the Quasi-Newton routines build up curvature information gradually as the search proceeds, relying only on the observed behavior of the objective function f and its gradient g.

In the Quasi-Newton approach, the inverse Hessian G^{-1} is approximated by a symmetric, positive-definite matrix H which is updated at each iteration. The kth iteration of the search has the form (Fletcher (1980)):

i) set $s^k = -H^k g^k$

ii) perform a line search along s^k, ending at

$$x^{k+1} = x^k + \alpha^k s^k$$

iii) update H^k, yielding H^{k+1}

where x^k denotes the kth solution vector, s^k represents the kth search direction, and α^k is a positive scalar.

Since the basic premise in the Quasi-Newton strategy is that curvature information can be estimated without explicitly forming the Hessian G, most research with these methods has focused on the development of improved formulae for updating H in step (iii) above. At the present, the BFGS (Broyden-Fletcher-Goldfarb-Shanno) update is generally regarded as the most effective. The update has the form:

$$H_{BFGS}^{k+1} = H + (1 + \frac{\gamma^T H \gamma}{\delta^T \gamma}) (\frac{\delta \delta^T}{\delta^T \gamma}) - (\frac{\delta \gamma^T H + H \gamma \delta^T}{\delta^T \gamma})$$

where:

$$\gamma^k = g^{k+1} - g^k$$
$$\delta^k = x^{k+1} - x^k = \alpha^k s^k$$

and where the superscript k has been dropped for convenience. Further details of the BFGS update and other aspects of the Quasi-Newton methods are provided, for example, by Broyden (1972), Dennis and Moré (1977) and Fletcher (1980).

8

To accomplish the line search in step (ii) above, a strategy outlined by Fletcher (1980) was employed, together with certain ad hoc modifications described by DeNatale (1982). This particular strategy permits the line search to be carried out to any degree of accuracy, which is ideal, since a primary advantage of the BFGS update is that it generally performs most efficiently with low accuracy line searches.

Because no analytic expressions are available for first derivatives g, gradients must be evaluated by finite differences. The strategy used is patterned after that described by Stewart (1967). Directed by the local geometry of the objective function, the algorithm switches between forward and central differencing formulae, and continually adjusts the differencing interval, in an attempt to balance round-off and truncation errors.

Finally, the code permits constraints to be imposed on the various constitutive parameters in the form of simple bounds:

$$\ell_i < x_i < u_i$$

where ℓ_i and u_i represent the minimum and maximum values that parameter x_i can assume. A restriction of this kind would be appropriate if either (i) there were certain theoretical restrictions placed on the value which a given parameter could assume, or (ii) there were certain ranges of parameter values beyond which the numerical implementation of the material model became unstable, or (iii) certain material properties were observed experimentally to vary over some finite range, and there was no overwhelming reason to fix them prior to calibration at any particular values.

There are a number of ways to directly (through the use of a constrained optimization code) or indirectly (through the use of barrier and penalty functions, Lagrange multipliers and/or variable transformations) account for simple bounds.

9

After a review of available literature, it was concluded that a trigonometric variable transformation would provide the most straightforward solution. Hence, rather than minimizing $f(x)$ subject to $\ell_i < x_i < u_i$, the program makes the transformation:

$$x_i = \ell_i + (u_i - \ell_i) \sin^2 y_i$$

or:

$$y_i = \sin^{-1} [(x_i - \ell_i)/(u_i - \ell_i)]^{1/2}$$

and minimizes $f[x(y)] = f(y)$, where y_i can now assume any value in the range $-\infty < y_i < +\infty$. Although variable transformations are potentially subject to a number of difficulties (as described, for example, by Gill et al (1981)), the above transformation was found to work extremely well in this application, with no detectable problems.

1.4) Generation of Model Predictions

The search for the optimal set of parameter values is directed by the Quasi-Newton strategy previously described. However, in order to evaluate the objective function at some location x, it is first necessary to generate a corresponding set of model predictions. To accomplish this, the calibration code relies on two subroutines — EVAL and CLAY — developed by Herrmann et al (1980, 1981) during previous research with the Bounding Surface model. Subroutine EVAL performs, essentially, single-element, incremental-iterative finite element analyses of bodies under a homogeneous state of stress and/or strain. Subroutine CLAY consists of a numerical implementation of the governing constitutive equations, and thus, when called, provides the appropriate material response to the given stress and/or strain increment. In order to adapt the present calibration code to other constitutive models, one need only replace CLAY with the

appropriate materials subroutine. Additional details of subroutines EVAL and CLAY are presented by Herrmann et al (1980, 1981, 1982).

1.5) The Calibration Data Base

The ultimate goal of the calibration process is to identify that set of parameter values which enables the theoretical model to most closely simulate the observed material response. This goal is ordinarily accomplished by fitting the model to a representative set of experimentally observed stress-strain relations or "calibration data base." Ideally, this calibration data base should be complete and diverse enough that all important aspects of the material's response are included, and all necessary constitutive parameters may be uniquely established.

In its most general form, the Bounding Surface formulation becomes a fully three-dimensional stress-strain model. With a single set of parameter values, the model may be applied to specimens at all overconsolidation ratios (OCR's), subjected to either monotonic or cyclic compression and/or extension loading, under either drained or undrained conditions. Hence, to establish the optimal values of the necessary constitutive parameters, the calibration data base should ideally contain observations (in the form of q vs p', q vs ϵ_1 and u vs ϵ_1 relations) from the following seven standard laboratory tests:

i)   an isotropic (or K_o) consolidation or drained compression test, with both loading and unloading; and,

ii-vii)   undrained triaxial compression and extension tests on specimens in the normally (OCR = 1), lightly (1 < OCR < 2.5) and heavily (OCR > 4) overconsolidated regions.

The results of the consolidation test are required to establish the slopes of the isotropic consolidation and swell/recompression curves in e-ln p' space, λ and κ. These two parameters belong to the class of traditional material properties, and

11

would normally be assigned values immediately, prior to using the automated calibration procedure. The results of the six undrained triaxial experiments are required to determine the 12 model constants cited in section 1.1, and would thus provide the data needed to direct the automated calibration procedure. Naturally, if a less general form of the Bounding Surface model is acceptable, the number of constitutive parameters involved, and the number of laboratory experiments required, can be drastically reduced. For example, if the model is only to be applied to normally consolidated soils loaded in triaxial compression, the number of required constitutive parameters drops from 19 to 7, and only the isotropic consolidation results and a single triaxial test are needed for model calibration.

Although the above data base is strongly recommended, the Bounding Surface model could also be calibrated using other types of data. For example, drained rather than undrained tests could be employed. However, undrained tests are preferable, since good initial estimates for many of the model parameters can be made by examining the experimentally observed undrained stress paths.

There is also some evidence that the calibration data base need not necessarily include data from all three regions of overconsolidation ratio (see DeNatale (1982)). That is, it may be sufficient to include only tests from the normal and heavy ranges, or, perhaps, even from the heavy range alone. The data which supports this possibility is not, however, conclusive, and therefore testing at all three overconsolidation regions is still advised.

In addition, the experimental observations need not necessarily include all three response relations q vs p', q vs ϵ_1 and u vs ϵ_1. Note that of the four undrained response parameters p', q, u and ϵ_1, only three are independent. In practice, p' is never actually measured, but, rather, is computed from the relation:

12

$$p' = \frac{3(\sigma_{3c} - u) + q}{3}$$

where σ_{3c} represents the applied lateral confining pressure. Thus, any two of the three relations cited above will completely define the soil response. The use of q vs p' or q vs ε_1 data alone is insufficient, since each of these relations is insensitive to certain of the constitutive parameters. There is some evidence that the use of u vs ε_1 data alone may, however, be adequate (see DeNatale (1982)). Nevertheless, the use of all three response relations appears to increase the rapidity with which the optimization algorithm converges to the minimum. Presumably, the inclusion of redundant data reinforces the correct search direction. Hence, since the cost of a computer directed calibration run is only marginally affected by the number of response relations included in the calibration data base, it is therefore still recommended that all three of the relations cited above be used.

Finally, it may be possible to use testing devices other than the triaxial apparatus to acquire the necessary experimental observations. Although the conventional triaxial apparatus is the most common and versatile laboratory device, the simple shear apparatus could, for example, also be used. In general, the material's observed stress-strain characteristics will, to some extent, be dependent on the testing device employed. Thus, in practical problems, the laboratory device used to acquire the calibration data base should simulate, as closely as possible, the loading conditions for which Bounding Surface predictions will eventually be generated.

1.6) Practical Considerations

As previously mentioned in section 1.2, the automated calibration code seeks to locate the objective function's global minimum. Unfortunately, there is no guarantee that the algorithm will always succeed. The Quasi-Newton

strategy employed by the model calibration code, like most, if not all, practical optimization algorithms, is designed only to locate local minima in the vicinity of the initial estimates. Hence, the probability that the true global minimum will be found is directly related to the degree of unimodality exhibited by the objective function and the accuracy of the initial starting guess.

Preliminary research by DeNatale (1982) has shown that the use of the absolute-Euclidean measure of error leads to a more unimodal, and thus desireable, objective function. A procedure for acquiring improved starting estimates has also been developed by DeNatale (1982). Through actual testing with a number of different soils, this strategy has been found to produce starting estimates which enable the automated calibration code to consistently locate the global minimum. In practice, however, the only way to ensure that the global minimum has been found is to conduct the search from a variety of different starting points. The solution which yields the lowest value of the objective function may then be regarded as the true global minimum.

A second practical consideration concerns the quality of the calibration data base. The user should ensure that the experimental observations included in the calibration data base are diverse enough to permit the optimal values of the required unknown model parameters to be uniquely defined. For example, if the code is used to identify those model parameters associated with the heavily overconsolidated material response, the calibration data base must include observations made on heavily overconsolidated soil specimens. If the necessary experimental data is not included, the program will continue to execute, but the final computed "optimal" values of the undefined parameters will be very close to the initial estimates.

The major consequence of an inadequate or incomplete calibration data base is therefore related to the cost of the analysis. Certain computational

14

costs increase in proportion to n^2 (where n represents the number of parameters whose optimal values are being sought), and a single gradient evaluation requires either n or 2n additional objective function evaluations, depending on whether forward or central differencing formulae are used. Thus, to minimize the cost of the analysis, the user should seek to identify only those parameters whose optimal values can be defined, given the particular experimental data base. A comprehensive discussion of the influence of each of the 19 model parameters is provided by DeNatale (1982), which may be referred to if any uncertainty exists.

1.7) Verification

In order to verify the viability of the new computer aided calibration procedure, the method was applied to a number of representative data bases, both artificial and real. The outcome of these studies is discussed in detail by DeNatale (1982). Among the real data bases to which the automated process was applied was the experimental research on Kaolin reported by Wroth and Loudon (1967). The Bounding Surface model was calibrated on the basis of undrained triaxial compression tests on samples at overconsolidation ratios of OCR = 1.0, 1.5 and 4.5. With the necessary constitutive parameters having thus been fixed at their optimal values, predictions were then generated for (i) undrained triaxial compression tests at OCR = 1.2, 1.8, 2.5, 3.0 and 6.5, (ii) an undrained cyclic triaxial compression test at OCR = 1.0, and (iii) a drained triaxial compression test at OCR = 1.3. The experimental observations and model predictions are compared in Figures 1.1 through 1.3. Further details of the calibration procedure and predictions for this particular data base are provided by both DeNatale (1982) and Herrmann et al (1982).

15
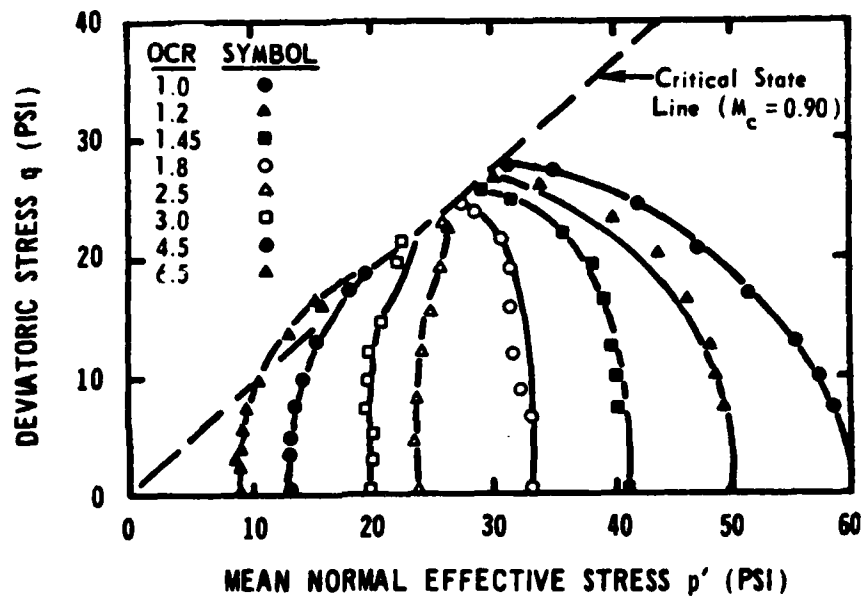
Figure 1.1: Experimental Observations and Model Predictions For the Undrained Triaxial Compression Tests
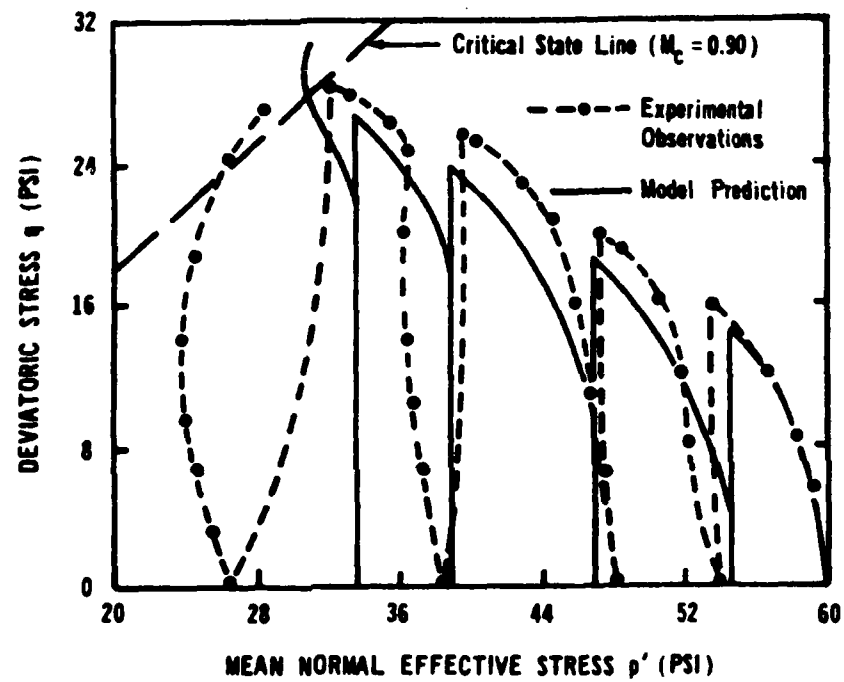
Figure 1.2: Experimental Observations and Model Predictions For The Undrained Cyclic Triaxial Compression Test (OCR = 1.0)
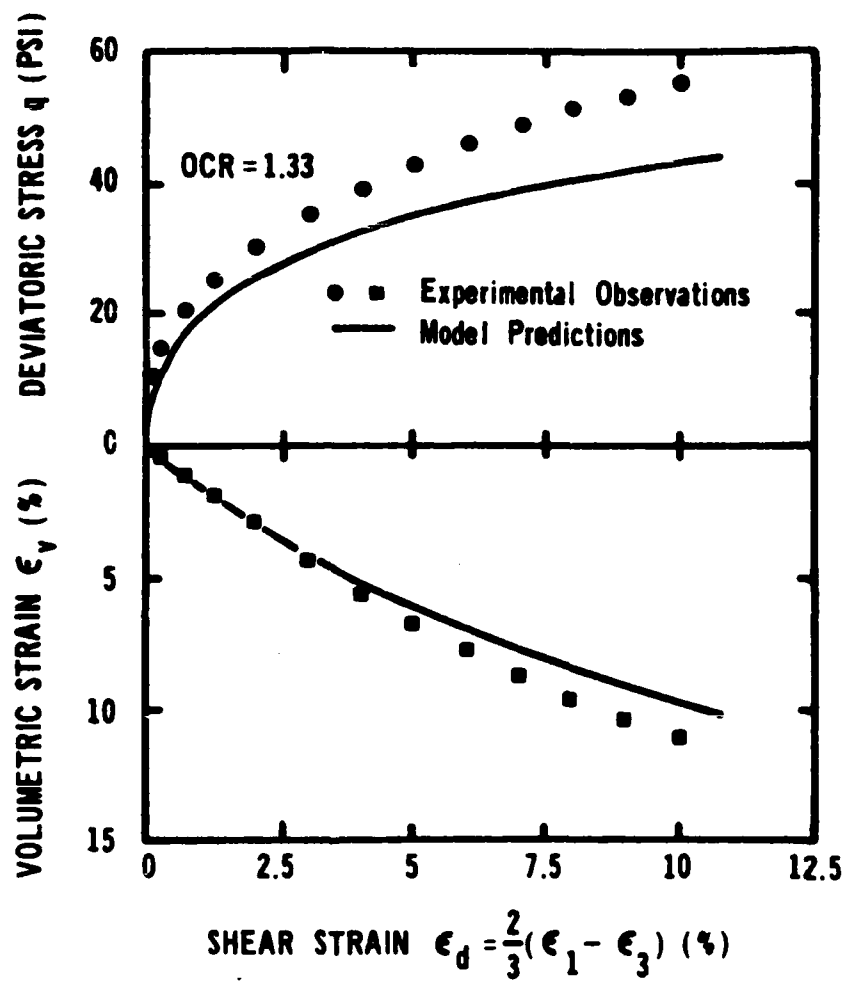
Figure 1.3: Experimental Observations And Model Predictions For The Drained Triaxial Compression Test (OCR = 1.3)

18

1.8) Cost

The automated calibration code has been written in FORTRAN and implemented on both an LSI-11/23 minicomputer, as well as a VAX-11/780 super-minicomputer. The cost of a given analysis is controlled primarily by the number of distinct experimental tests included in the calibration data base and the number of constitutive parameters whose optimal values are being sought. A typical computer calibration, such as that reported in section 1.7 for the data of Wroth and Loudon (1967), requires from 200-400 objective function evaluations, or about 30-60 minutes of VAX CPU time, at a cost of approximately $25.00-$50.00. When compared to the expense associated with the acquisition of the experimental calibration data base, or the expense of subsequent finite element analyses involving the calibrated model, the cost of a typical computer aided calibration becomes relatively low. Note also that the material model need only be calibrated once for any particular soil, regardless of the variety and number of finite element analyses that may subsequently be performed.

1.9) Conclusion

In order to facilitate the calibration of sophisticated constitutive models, an automated FORTRAN code has been developed and tested. The code employs a Quasi-Newton optimization strategy to locate that set of parameter values which minimizes the weighted residual between the model predictions and the experimental calibration data base. Through application to a number of real soils, this new procedure has been found to be an efficient, reliable and economical means of accomplishing model calibration.

Although the code was developed specifically for use with the Bounding Surface plasticity model, it can be readily adapted to other constitutive formulations. Because the code greatly reduces the dependence of calibration

success on user expertise, it significantly increases the accessibility and usefulness of sophisticated material models to the general engineering community.

In addition to being a useful practical tool, the code can also be of great value to the theoretician involved in model development. Since use of the code enables various sets of model predictions to be quantitatively compared, it enables one to investigate such features as (i) the model's sensitivity to each of the constitutive parameters, (ii) the significance and uniqueness of each parameter's effect, (iii) the influence of the calibration data base on the computed optimal solution and (iv) the uniqueness of a given solution or set of model predictions. These uses of the code, as well as several others, are discussed and illustrated in greater detail by DeNatale (1982).

The computer aided procedure is not, however, a cure-all, and use of the automated scheme does not guarantee that all problems associated with manual model calibration will disappear. Many of the difficulties encountered during the manual calibration of a particular material model have little or nothing to do with user expertise, but, rather, arise due to the nature of the formulation itself. The presence, in a particular model, of an excessive number of constitutive parameters, or ill-defined parameters with non-unique effects, will always make a unique solution difficult, or perhaps impossible, to find, regardless of whether a computer approach is used or not. Thus, it is best to regard the automated procedure as a useful tool for well-constructed models, rather than as a fool-proof cure for ill-constructed ones.

II. INPUT INSTRUCTIONS FOR MODCAL

Bounding Surface Soil Plasticity Model Calibration

and Prediction Code

by

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Department of Civil Engineering

University of California, Davis

November, 1982

II. INPUT INSTRUCTIONS FOR MODCAL

I. Heading Information Card:

1st Card (20A4):

Columns

1 - 80   **TITL**   :   any information that is to be printed as a title for the analysis

II. Control Codes and Analytical Options Card:

1st Card (16I5):

Columns

1 - 5   **NTST**   :   number of distinct experimental tests for which model predictions are desired, or on which model calibration is to be based

6 - 10   **JOPT**   = 0 → model calibration not required
                              1 → model calibration required

11 - 15   **JRUN**   = 0 → model predictions not generated
                              1 → model predictions generated

16 - 20   **JPLT**   = 0 → observed and predicted responses not plotted
                              1 → observed and predicted responses plotted

21 - 25   **KRSL**   = 0 → Euclidean measure of error used
                              1 → vertical measure of error used

26 - 30   **KNRM**   = 0 → absolute residuals used
                              1 → squared residuals used

31 - 35   **NOPT**   :   number of parameters to be established by optimization

36 - 80   **KOPT (II), II** = 1, **NOPT**:   property numbers of the NOPT parameters to be established by optimization (see section III and Table 2.1). Use I5 formats and continue on a 2nd card if necessary (that is, if NOPT > 9)

III. **Material and Model Properties Cards:**[†]

1st Card (8E10.3):

Columns

1 - 10	λ	:	slope of the isotropic consolidation line in e - ln p space
11 - 20	κ	:	slope of the swell/recompression line in e - ln p space
21 - 30	M_c	:	slope of the critical state line in triaxial compression
31 - 40	M_e/M_c	:	ratio of extension to compression values
41 - 50	G	:	elastic shear modulus (or, alternatively, Poisson's ratio ν)
51 - 60	Γ	:	combined bulk modulus of the pore water and soil skeleton. For drained conditions, $\Gamma = 0$. For undrained conditions, a value in the range $\Gamma = (10^4$ to $10^6) \cdot p_a$ is recommended
61 - 70	p_ℓ	:	transitional stress at which the drained compression response changes from linear in e - p space to linear in e - ln p space. If no data in this region is available, a value in the range $p_\ell = (0.3$ to $1.0) \cdot p_a$ is recommended
71 - 80	p_a	:	atmospheric pressure

2nd Card (7E10.3):

Columns

1 - 10	R_c	:	surface shape parameter associated with ellipse 1 in compression
11 - 20	A_c	:	surface shape parameter associated with the hyperbola in compression

---

[†] If model calibration is required, it is not necessary to enter estimates for those parameters whose optimal values are being sought.

21 - 30	T	:	surface shape parameter associated with ellipse 2 in compression and extension. A value in the range $0.05 < T < 0.15$ is suggested
31 - 40	R_e/R_c	:	ratio of extension to compression values
41 - 50	A_e/A_c	:	ratio of extension to compression values
51 - 60	c	:	projection point parameter. A value of $c = 0.0$ places the projection point at the origin
61 - 70	s	:	elastic nucleus parameter. A value of $s = 1.0$ causes the elastic zone to degenerate to a point

3rd Card (4E10.3):

Columns

1 - 10	h_c	:	primary hardening parameter in compression
11 - 20	m_c	:	secondary hardening parameter in compression. A value in the range $0.2 < m < 1.0$ is suggested
21 - 30	h_e/h_c	:	ratio of extension to compression values
31 - 40	m_e/m_c	:	ratio of extension to compression values

4th Cards (I5, 5X, 4E10.3):

One card is required for each of the NTST distinct experimental tests for which model predictions are desired:

Columns

1 - 5	ITST	:	test number
11 - 20	e_o	:	initial void ratio
21 - 30	P_o	:	initial maximum past effective preconsolidation pressure
31 - 40	σ_{co}	:	initial confining pressure
41 - 50	OCR	:	initial overconsolidation ratio, as defined by $OCR = P_o/\sigma_{co}$

24

IV.   Convergence Criteria and Iteration Information Card:

1st Card (3I5, 5X, 4E10.3):

<u>Columns</u>

1 - 5	KIND	=	0 → reformulated nearly incompressible analysis
			1 → non-reformulated nearly incompressible analysis

6 - 10   ITMX   :   maximum number of iterations permitted for a given solution increment. Values in the range 5 < ITMX < 10 are suggested

11 - 15   LARG   =   0 → engineering stresses and strains assumed

          1 → true stresses and natural (logarithmic) strains assumed

21 - 30   CNFR   :   establishes CNFR and 1/CNFR as the lower and upper limits for the calculated values of the Aitken's acceleration factors. Values in the range 0.0< CNFR < 1.0 are permitted, and if CNFR = 1.0, acceleration factors will not be used

31 - 40   ERMX   :   maximum allowable relative difference between the norms of the incremental stress and strain vectors from two consecutive iterations. Values in the range 0.001 < ERMX < 0.01 are recommended. If convergence does not occur within ITMX iterations, a message is printed and program execution will cease

41 - 50   PLIM   :   percentage of the maximum absolute experimentally observed y-value for a given plot, below which absolute rather than relative errors are used. Values in the range 0.0 < PLIM < 1.0 are permitted, with PLIM = 0.0 resulting in relative errors being used at all points and PLIM = 1.0 resulting in absolute errors being used at all points

51 - 60   TLIM   :   percentage of the maximum absolute experimentally observed y-value for a given type of relation, below which absolute rather than relative errors are used. Values in the range 0.0 < TLIM < 1.0 are permitted, with TLIM = 0.0 resulting in relative errors being used for all plots and TLIM = 1.0 resulting in absolute errors being used for all plots

25

V. Specified Loading History Cards:

The following sequence of cards must be supplied for each of the NTST distinct experimental tests for which model predictions are required:

1st Card (2I5):

Columns

1 - 5	ITST	:	test number
6 - 10	NSEG	:	number of distinct history segments into which the test is subdivided

2nd Cards (6 (I1, E9.2), 5X, I5, E10.3):

One card is required for each of the NSEG distinct history segments into which the test is subdivided:

Columns

1     $LTYP_1$ = 0 → σ_x is specified
                        1 → ϵ_x is specified

2 - 10 $VALU_1$:   value of σ_x or ϵ_x at the end of the given history segment

11      $LTYP_2$ = 0 → σ_y is specified
                        1 → ϵ_y is specified

12 - 20 $VALU_2$:   value of σ_y or ϵ_y at the end of the given history segment

21      $LTYP_3$ = 0 → σ_z is specified
                        1 → ϵ_z is specified

22 - 30 $VALU_3$:   value of σ_z or ϵ_z at the end of the given history segment

31      $LTYP_4$ = 0 → τ_{xy} is specified
                        1 → γ_{xy} is specified

32 - 40 $VALU_4$:   value of τ_{xy} or γ_{xy} at the end of the given history segment

41      $LTYP_5$ = 0 → τ_{xz} is specified
                        1 → γ_{xz} is specified

42 - 50 $VALU_5$:   value of τ_{xz} or γ_{xz} at the end of the given history segment

51	$LTYP_6$	=	0 $\rightarrow$ τ_{yz} is specified
			1 $\rightarrow$ γ_{yz} is specified
52 - 60	$VALU_6$	:	value of τ_{yz} or γ_{yz} at the end of the given history segment
66 - 70	NINC	:	number of increments into which the given history segment is to be subdivided
71 - 80	SRAT	:	incrementing ratio which defines the relative magnitude of two consecutive loading increments within the given history segment. A value of SRAT = 1.0 results in NINC equally spaced loading increments. By definition SRAT = $\Delta\sigma^n/\Delta\sigma^{n-1}$ (or $\Delta\epsilon^n/\Delta\epsilon^{n-1}$), where the superscripts n-1 and n denote the values in two consecutive loading increments

VI. Experimental Data and Plotting Instructions Cards:

The following sequence of cards must be supplied for each of the NTST distinct experimental tests for which model predictions are required:

1st Card (2I5, E10.3, I5):

Columns

1 - 5	ITST	:	test number
6 - 10	NPLT	:	number of distinct experimental relations associated with the given test
11 - 20	WTST	:	weighting factor for the given test. By default, WTST = 1.0
21 - 25	NWPT	:	number of distinct experimental observations associated with the given test to which weights other than 1.0 will be assigned

2nd Card (4 (2I5, E10.3)):

Columns

1 - 5	$KPLT_1$	:	type of plot for the first experimental relation (see Table 2.2)
6 - 10	$NEXP_1$	:	number of distinct experimental observations associated with the given relation
11 - 20	$WPLT_1$	:	weighting factor for the given relation. By default, WPLT = 1.0
21 - 80	→ →	:	repeat entries for columns 1-20 as described above (in the identical format) until all NPLT distinct experimental relations have been described

The following sequence of cards must be provided for each of the NPLT distinct experimental relations associated with test ITST for which NEXP ≠ 0:

3rd Cards (8E10.3):

Use as many cards as necessary to enter the y - values (or ordinates) of the NEXP distinct experimental observations associated with relation KPLT of test ITST

4th Cards (8E10.3):

Use as many cards as necessary to enter the x-values (or abscissae) of the NEXP distinct experimental observations associated with relation KPLT of test ITST

VII. Special Experimental Weightings Cards: (include <u>only</u> if $\sum_{i=1}^{NTST} NWPT_i \neq$ 0 in section VI):

<u>1st Cards (2I3, I4, E10.3):</u>

<u>Columns</u>

1 - 3	IT	:	test number
4 - 6	IP	:	plot number
7 - 10	IE	:	experimental observation number
11 - 20	WPNT	:	weighting factor for the IE^{th} observation in the IP^{th} plot of the IT^{th} test
21 - 80	➔ ➔	:	repeat entries for columns 1-20 as described above (in the identical format) until all $\sum NWPT$ distinct special weightings have been described. Continue on a 2nd card if necessary

The remaining input cards are only necessary for those analyses where JOPT = 1 (in section II) and, therefore, model calibration is required.

VIII. Calibration Control Codes Card:

1st Card (9I5):

Columns

Columns			
1 - 5	**NDIM**	:	number of parameters whose optimal values are being sought
6 - 10	**NFMX**	:	maximum allowable number of objective function evaluations
11 - 15	**IPRN**	=	1 → only final results printed 2 → results printed after each line search 3 → results printed after each objective function evaluation
16 - 20	**ICON**	=	1 → convergence based on parameter values 2 → convergence based on function values 3 → convergence based on gradient values
21 - 25	**ICUB**	=	0 → strict line search convergence criterion used 1 → normal line search convergence criterion used
26 - 30	**IHES**	=	0 → Hessian conditioning not performed 1 → Hessian conditioning performed
31 - 35	**NHES**	:	number of non-zero values to be entered for the initial inverse Hessian matrix. If NHES = 0, the inverse Hessian will initially be set to the identity matrix
36 - 40	**ISCA**	=	0 → parameter scaling not performed 1 → parameter scaling performed
41 - 45	**NSLN**	=	0 → no initial estimates computed 1 → initial estimates computed for either R_c or R_e 2 → initial estimates computed for both R_c and R_e

IX. Convergence Specifications Card:

1st Card (7E10.3, I5):

Columns

1 - 10	**EPFA**	:	convergence criterion associated with the objective function, f. If ICON = 2 (see section VIII), the search will be terminated when the k-th line search has failed to reduce f by an appreciable amount:

$$EPFA > (f_{k-1} - f_k)$$

11 - 20	**EPSA**	:	convergence criterion associated with the gradient of the objective function, g. If ICON = 3 (see section VIII), the search will be terminated when the slope of the k-th computed search direction is suitably small:

$$EPSA > (-g_k^T H_k g_k)$$

21 - 30	**C1**	:	value of the line search exclusion parameter α. A value of $\sigma \simeq 0.9$ is recommended
31 - 40	**C2**	:	value of the line search exclusion parameter ρ. A value in the range $0.0001 < \rho < 0.01$ is suggested
41 - 50	**C3**	:	value of the bracket check parameter τ. A value of $\tau = 0.1$ is recommended
51 - 60	**EPXL**	:	line search exit criterion associated with the parameters x. A value in the range $0.001 < EPXL < 0.01$ is suggested
61 - 70	**EPFL**	:	global fail-safe exit criterion associated with the objective function f
71 - 75	**NFLX**	:	maximum allowable number of objective function evaluations that may be made before the fail-safe termination criterion (based on EPFL) is invoked

X.   Finite-Difference Specifications Card:

1st Card (5E10.3):

Columns

1 - 10	ETAF	:	relative error associated with the computation of objective function values
11 - 20	ETAX	:	relative error associated with the representation of parameter values in a finite word-length machine
21 - 30	DMIN	:	smallest differencing interval permitted in calculating first derivatives by finite differences
31 - 40	FDER	:	maximum permissible relative truncation error. If the estimated error is greater than FDER, central rather than forward differences are used to compute the gradient. A value in the range 0.0001 < FDER < 0.01 is suggested
41 - 50	SFUN	:	scaling factor for the objective function. The optimization code seeks the minimum of SFUN•f. By default, SFUN = 1.0

XI.   Initial Estimates Card:

1st Cards (8E10.3):

Use as many cards as necessary to enter initial estimates for the NDIM parameters whose optimal values are being sought. For typical values, see Table 2.3

XII.   Lower Bounds Card:

1st Cards (8E10.3):

Use as many cards as necessary to specify the lower bounds (ℓ_i < x_i) associated with each of the NDIM parameters whose optimal values are being sought. For typical values, see Table 2.3

XIII.   Upper Bounds Card:

1st Cards (8E10.3):

Use as many cards as necessary to specify the upper bounds (x_i < u_i) associated with each of the NDIM parameters whose optimal values are being sought. For typical values, see Table 2.3

32

XIV. Initial Differencing Intervals Card:

1st Cards (8E10.3):

Use as many cards as necessary to specify the initial finite difference differencing intervals associated with each of the NDIM parameters whose optimal values are being sought. For typical values, see Table 2.3

XV. Scaling Factors Card: (include only if ISCA = 1 in section VIII):[†]

1st Card (8E10.3):

Use as many cards as necessary to specify the scaling factors associated with each of the NDIM parameters whose optimal values are being sought

XVI. Convergence Specifications Card: (include only if ICON = 1 in section VIII):

1st Card (8E10.3):

Use as many cards as necessary to specify the convergence criteria associated with the parameters x. If ICON = 1, the search will be terminated when the kth line search has established the location of the minimum to a sufficiently high degree of accuracy:

$$\epsilon_1 > |x_i^{k-1} - x_i^k| \qquad (i=1, \text{NDIM})$$

33

XVII. Initial Inverse Hessian Specification Card: (include <u>only</u> if NHES > 0 in section VIII):[†]

<u>1st Card (4 (2I5, E10.3))</u>:

<u>Columns</u>

1 - 5	**IR**	:	row number
6 - 10	**IC**	:	column number
11 - 20	**H2IJ**	:	value of the $(IR, IC)^{th}$ component of initial inverse Hessian matrix
21 - 80	➜ ➜	:	repeat entries for columns 1-20 as described above (in the identical format) until all NHES components have been specified. Continue on a 2nd card if necessary

XVIII. Analytic Initial Estimates Card: (include <u>only</u> if NSLN > 0 in section VII):

The following card must be repeated NSLN times — once for each initial estimate required:

<u>1st Card (5E10.3, I5)</u>:

<u>Columns</u>

1 - 10	λ	:	slope of the isotropic consolidation line in e - ln p space
11 - 20	κ	:	slope of the swell/recompression line in e - ln p space
21 - 30	**PCON**	:	initial value of the mean normal effective stress $p' = p'_o$
31 - 40	**PFAL**	:	critical value of the mean normal effective stress $p' = p'_f$
41 - 50	**RI**	:	initial estimate for the surface shape parameter R. By default, RI = 2.50
51 - 55	**KR**	:	$1 \rightarrow R = R_c$ is assumed $2 \rightarrow R = R_e^c$ is assumed

---

[†] For most analyses, ISCA = 0 and NHES = 0, and therefore input cards XV and XVII will normally be omitted.

Figure 2.1: A Schematic Illustration Of The Bounding Surface and Radial Mapping Rule In Invariant Stress Space

TABLE 2.1

Correspondence Between Parameter Type and Property Number

Property Number	Parameter Type	Property Number	Parameter Type
1	λ	11	T
2	κ	12	R_e/R_c
3	M_c	13	A_e/A_c
4	M_e/M_c	14	c
5	G	15	s
6	Γ	16	h_c
7	P_ℓ	17	m_c
8	P_a	18	h_e/h_c
9	R_c	19	m_e/m_c
10	A_c		

TABLE 2.2

Correspondence Between Relation Type and Relation Number

Relation Number	Relation Type
1	q vs $p^\ast$
2	q vs ϵ_1
3	$p^\ast$ vs ϵ_1
4	u vs ϵ_1
5	ϵ_v vs ϵ_1

TABLE 2.3

Typical Parameter Values, Bounds and Initial Differencing Intervals

Parameter Number	Parameter	Typical Value	Typical Lower Bound	Typical Upper Bound	Typical Initial Differencing Interval
1	λ	0.20	0.10	0.40	0.02
2	κ	0.04	0.02	0.08	0.004
3	M_c	1.00	0.75	1.25	0.10
4	M_e/M_c	1.00	0.75	1.25	0.05
5	G	2000	1000	10000	200
7	P_ℓ	P_a	$0.25 \cdot P_a$	P_a	$-0.25 \cdot P_a$
9	R_c	2.50	2.00	3.00	0.05
10	A_c	0.10	0.03	0.20	0.01
11	T	0.10	0.05	0.15	0.05
12	R_e/R_c	1.00	0.75	1.25	0.02
13	A_e/A_c	1.00	0.50	2.00	0.10
14	c	0.50	0.00	0.75	0.05
15	s	1.00	1.00	2.00	0.50
16	h_c	0.25	0.05	2.00	0.05
17	m_c	0.50	0.20	1.00	0.50
18	h_e/h_c	1.00	0.50	4.00	0.20
19	m_e/m_c	1.00	0.50	4.00	1.00

III. EXPLANATORY NOTES AND INPUT RECOMMENDATIONS

3.1) Input Sections I and II

The program MODCAL may be used to accomplish model calibration (JOPT=1), or to generate model predictions (JRUN=1), or both. If the code is used to generate model predictions, plots of the experimentally observed and theoretically computed response relations may be requested (JPLT=1) to supplement the numerical output data.

The constants KRSL and KNRM partially define the manner by which the objective function is formed (see section 1.2). If model calibration is not required, the values of KRSL, KNRM, NOPT and KOPT(II) may be omitted. The absolute-Euclidean measure of error (KRSL=0 and KNRM=0) has been found to produce the most easily minimized objective function. Hence, in most applications this measure should be employed. The vertical measure of error (KRSL=1) should only be used if all of the response relations included in the calibration data base (see section 3.4) are either monotonically increasing or monotonically decreasing with respect to their x-values or abscissae. For example, the vertical measure could be used with curves a-g of Figure 3.1, but not with curve h (since this relation first increases and then decreases with respect to its x-coordinates). Thus, the vertical measure should normally not be used if q vs p' relations are included in the calibration data base.

The NOPT parameters to be established by optimization are identified by entering their respective "property numbers" in the KOPT(II) array. The correspondence between parameter type and property number is defined in Table 2.1. As may be seen, the ordering scheme used in Table 2.1 is identical to that employed in input section III; that is, λ is property 1, κ is property 2, and so forth.
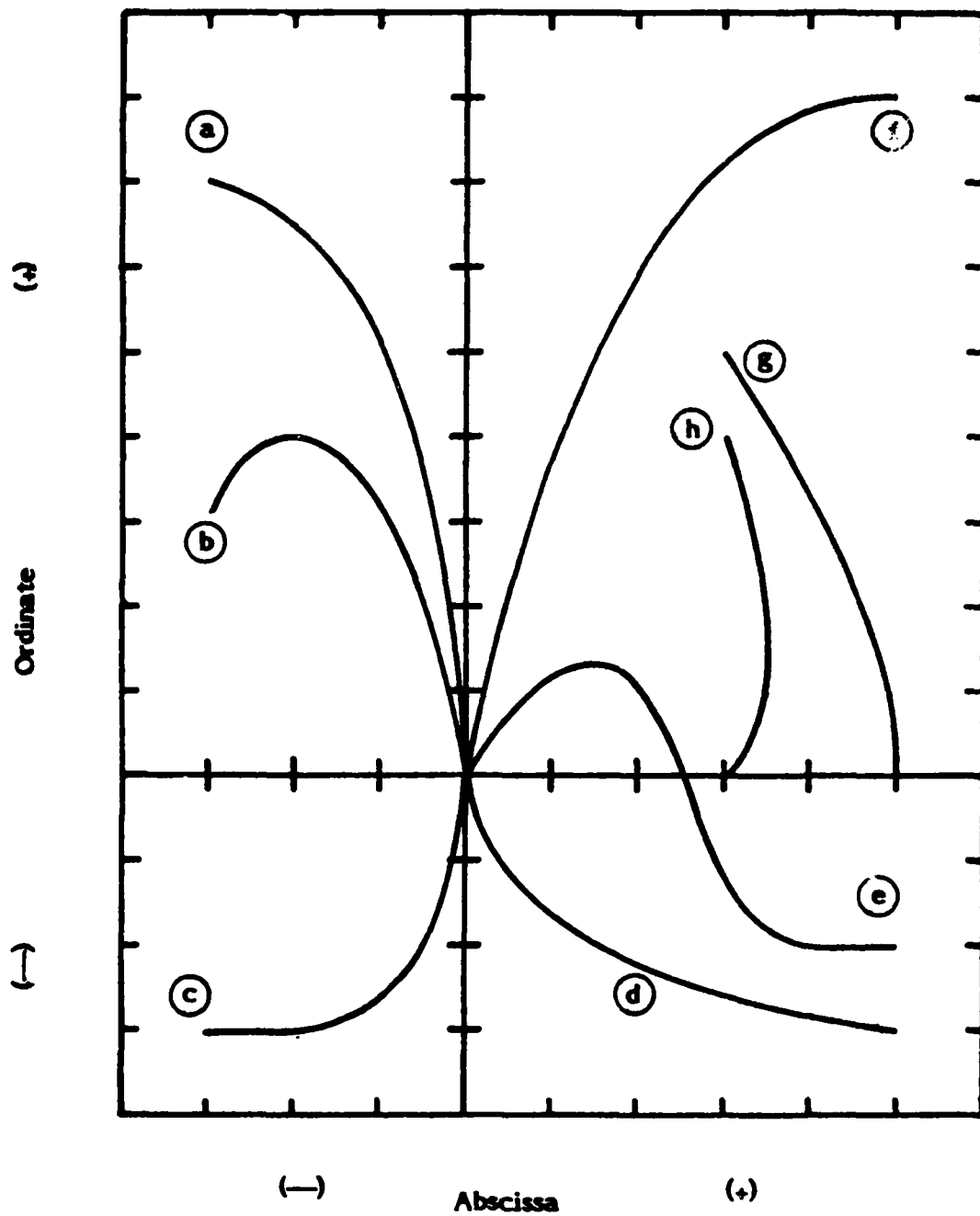
Figure 3.1: A Qualitative Representation of Some Typical Soil Response Relations

39

3.2) Input Section III

The values of the required material and model properties are specified in input section III. A more detailed description of the various parameters, including an examination of their qualitative and quantitative influence on the predicted material response, is presented by DeNatale (1982). If model calibration is required, it is not necessary to enter estimates for those parameters whose optimal values are being sought.

3.3) Input Sections IV and V

As mentioned earlier in section 1.4, the code relies on two subroutines —EVAL and CLAY — to generate the required model predictions. These predictions can be acquired by means of either a "conventional" (KIND=1) or a "reformulated" (KIND=0) analysis. The reformulated analysis is a modification of the mixed finite element procedure developed by Herrmann (1965) for incompressible and nearly incompressible solids. In it, the six strain components together with the pore water pressure represent the "mixed" set of primary dependent variables. When both the conventional and reformulated analyses converge, they yield identical results. However, for certain undrained and near-failure conditions the reformulated analysis may successfully converge, while the conventional analysis may not. Additional details of the two formulations are provided by Herrmann et al (1981, 1982).

Both types of analyses use the method of successive approximation, and acceleration factors can be applied to the stress and strain vectors to increase the rate of convergence. The acceleration factors are calculated by means of the Aitken's ∇^2 formula (see, for example, Isaacson and Keller (1966)), and the maximum and minimum factor values are controlled by the constant CNFR. By default, CNFR=0.3, and if CNFR=1.0, acceleration factors will not be used.

40

Finally, the analyses may be accomplished using either engineering (LARG=0) or natural (LARG=1) strains. This option was introduced solely to facilitate the comparison of model predictions with experimental observations reported in either format.

In order to simulate a given experimental test ITST, the applied loading history must be specified in piecewise form by entering descriptions of its NSEG distinct "history segments." A particular history segment is defined by the values of the six stress and/or strain components ($LTYP_i$ and $VALU_i$; i=1,6) which act on the material element at the end of that segment. Within the program, the segment is further subdivided into NINC separate increments, and iteration is performed within each increment. The relative size of the increments which make up a given history segment is controlled by the constant SRAT. By default SRAT=1.0, and the segment is then subdivided into NINC equal sized increments. If, in any particular increment, convergence (as defined by the constant ERMX) is not achieved within ITMX iterations, an appropriate message is printed and the analysis will then stop. By default, ERMX=0.01 and ITMX=10.

A conventional monotonic triaxial loading could possibly be described with as few as one history segment. For example, a strain-controlled test which began at an all-around confining stress of $\sigma_{3c} = \sigma_o$ and ended, after 30 equal sized steps, at an axial strain of ϵ_1=15%, could be simulated by specifying:

$$\sigma_x = \sigma_o$$

$$\sigma_y = \sigma_o$$

$$\epsilon_z = 0.15$$

$$\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0.0$$

$$NINC = 30$$

$$SRAT = 1.0$$

41

There are three situations where it would be necessary to use more than one segment to describe a particular loading history; namely:

i) if one or more of the six loading components changes during the test from a stress quantity to a strain quantity (or vice versa);

ii) if the loading history cannot be subdivided into increments of the desired length through the use of the parameters NINC and SRAT, alone; or,

iii) if a cyclic loading history must be described.

For example, a minimum of two segments would be required if the triaxial specimen previously described was first loaded to an axial strain of $\epsilon_z = 15\%$ and then unloaded to its initial stress state $\sigma_{3c} = \sigma_o$ in 10 equal sized steps:

(i) $\sigma_x = \sigma_o$ (ii) $\sigma_x = \sigma_o$

$\sigma_y = \sigma_o$ $\sigma_y = \sigma_o$

$\epsilon_z = 0.15$ $\sigma_z = \sigma_o$

$\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0.0$ $\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0.0$

NINC = 30   NINC = 10

SRAT = 1.0   SRAT = 1.0

Or, if only loading was involved, but it was desired to use increments of $\Delta\epsilon_z = 0.2\%$ for axial strains of $\epsilon_z < 2\%$ and increments of $\Delta\epsilon_z = 1.0\%$ for axial strains of $\epsilon_z > 2\%$, a minimum of two segments would again be required:

(i) $\sigma_x = \sigma_o$ (ii) $\sigma_x = \sigma_o$

$\sigma_y = \sigma_o$ $\sigma_y = \sigma_o$

$\epsilon_z = 0.02$ $\epsilon_z = 0.15$

$\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0.0$ $\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0.0$

NINC = 10   NINC = 13

SRAT = 1.0   SRAT = 1.0

Careful thought should be given to the total number of increments used to represent a particular loading history. The importance of increment size in EVAL-directed analyses is analogous to the significance of grid size in a general finite element analysis; that is, the predicted material response may, to some extent, be influenced by the size of the increments into which the applied loading history is subdivided. If too few increments are used, convergence may not occur, or the program may converge to an "incorrect" state of stress and/or strain. If too many increments are used, the analysis would become unnecessarily expensive. Note that the cost of a given analysis is controlled not by the number of history segments or loading increments used, but, rather, by the total number of iterations required, all increments considered. Thus, an analysis which uses a greater number of increments may actually be more economical, if it can be completed in a lesser total number of iterations.

Typically, from 20-40 increments are sufficient to describe a particular monotonic loading. In general, undrained analyses require smaller increments than drained analyses, and the unloading portions of a test can normally be described with larger increments than the loading segments. The code is designed to simulate tests which begin at a hydrostatic state of stress ($\sqrt{J_2}=0$), and relatively small increments should be used if and when the stress state first moves off the hydrostatic (I_1) axis. An automated increment generation scheme is currently under development as part of the ongoing research with the Bounding Surface model. However, since the scheme has not yet been incorporated into the computer code, some user-initiated checks should first be made with respect to optimal increment size, before proceeding with any potentially expensive analyses.

The parameters PLIM and TLIM are the second group of constants associated with the formation of the objective function. Their roles can perhaps

43

best be understood by considering the expression for residuals used in the code. The unweighted residual R_k corresponding to experimental observation k is given by:

$$R_k = [\frac{r_k}{y_1} \cdot \frac{y_p}{y_2}]$$

where:

If $(\frac{y_k}{y_p} > PLIM)$, $\quad y_1 = y_k$

If $(\frac{y_k}{y_p} < PLIM)$, $\quad y_1 = y_p \cdot PLIM$

If $(\frac{y_p}{y_r} > TLIM)$, $\quad y_2 = y_p$

If $(\frac{y_p}{y_r} < TLIM)$, $\quad y_2 = y_r \cdot TLIM$

and where (refer also to Figure 3.2):

r_k = the normalized distance between the kth experimental observation and the prediction curve;

y_k = the y-value (or ordinate) of the kth experimental observation;

y_p = the maximum absolute experimentally observed y-value, all points of the given relation considered;

y_r = the maximum absolute experimentally observed y-value, all relations of the given type considered;

y_1 = the base y-value for residual scaling for all points of a given relation; and,

y_2 = the base y-value for residual scaling for all relations of a given type.
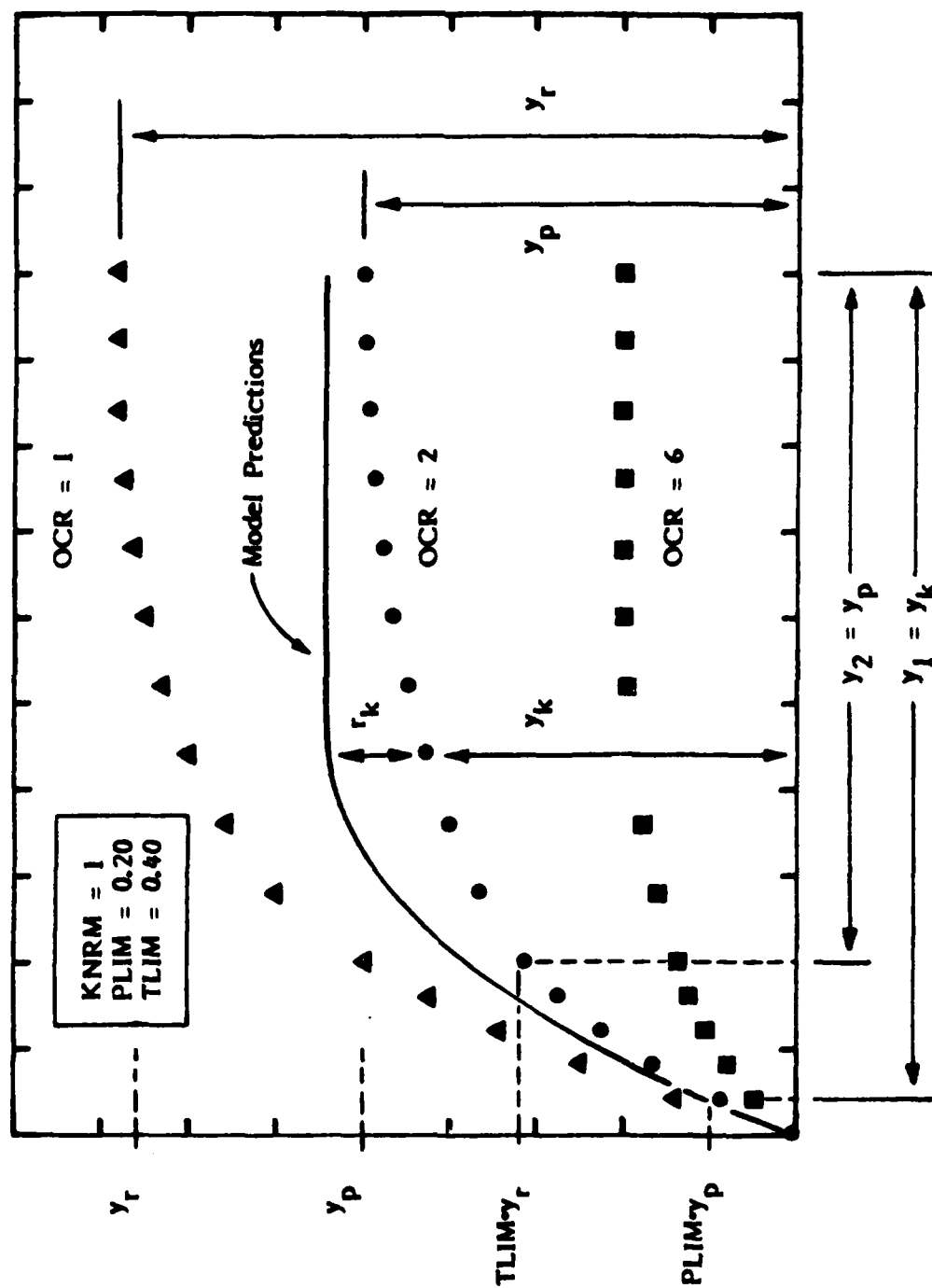
Figure 3.2: Definition Of Terms Associated With The Residual Scaling Parameters PLIM and TLIM (For The Experimental Observations At OCR = 2)

As pointed out earlier in section 1.2, before an objective function can be formed, the user must define to the code what is meant by "equal" errors. If equality is based on relative errors throughout, then:

$$PLIM = 0$$

$$TLIM = 0$$

$$y_1 = y_k$$

$$y_2 = y_p$$

$$R_k = \frac{r_k}{y_k} \cdot \frac{y_p}{y_p} = \frac{r_k}{y_k}$$

and therefore the residual R_k will be the same at two points A and B only if there are identical relative errors (r_k/y_k) at A and B. If equality is based on absolute errors throughout, then:

$$PLIM = 1$$

$$TLIM = 1$$

$$y_1 = y_p$$

$$y_2 = y_r$$

$$R_k = \frac{r_k}{y_p} \cdot \frac{y_p}{y_r} = \frac{r_k}{y_r}$$

and therefore the residual R_k will be the same at two points A and B only if there are identical absolute errors r_k at A and B. Additionally, equality could be based on relative errors within a given relation (PLIM=0) but absolute errors between relations of a given kind (TLIM=1), or vice versa (PLIM=1 and TLIM=0), or anywhere in between (0 < PLIM < 1 and 0 < TLIM < 1). By default, PLIM=0.01 and TLIM=0.01.

The most serious argument against the use of relative errors is that small deviations at low magnitude observations can produce extremely large contributions to the total computed overall error. In other words, as y_k approaches zero, the quantity (r_k/y_k) approaches infinity, regardless of the absolute error r_k. The constants PLIM and TLIM permit the use of relative errors over most of the data base, while preventing those observations whose magnitudes are relatively small from unduly influencing the computed overall "goodness" of the model predictions. The net effect of a given combination of PLIM and TLIM could probably be duplicated through a judiciously chosen set of weighting factors in input section VI (see section 3.4). However, weighting factors were introduced for a distinctly different reason, as will be considered in section 3.4.

3.4) Input Sections VI and VII

The type of plots associated with a given test ITST are identified by entering their respective "relation numbers" as KPLT. The correspondence between plot type and relation number is defined in Table 2.2. As may be seen, a q vs p´ plot is assigned a relation number of KPLT=1, and so forth.

If NEXP≠0, the individual data points associated with the experimental response relation KPLT must be entered in the same order in which they were originally observed, and not in some random fashion. In other words, the first point specified must correspond to the initial experimental measurement, while the last point entered must represent the final observation. This ordering scheme must hold for both the 3rd and 4th card group entries of input section VI. If NEXP=0, the 3rd and 4th cards associated with the particular relation KPLT must be omitted.

The weighting factors WTST, WPLT and WPNT are the third and final group of constants associated with the formation of the objective function. Like KRSL and KNRM (see section 3.1) and PLIM and TLIM (see section 3.3), their

47

values may be omitted if model calibration is not required. By default, all tests, relations and individual observations are weighted equally. However, different weights may be assigned to specific components of the data base if it is felt that certain tests, relations or observations are more reliable or representative than others, or if it is necessary to have the final model predictions fit some data more closely than others. Thus, by assigning a weight of WPNT=0 to a given experimental observation, it becomes possible to include that point in the calibration data base, without it influencing the computed value of the objective function.

If model calibration is required, the content of the calibration data base should be complete and diverse enough to permit each of the unknown parameters to be uniquely determined. The recommended calibration data base is described in detail in section 1.5, and the consequences of an inadequate or incomplete data base are considered in section 1.6.

3.5) Input Sections VIII and IX

As mentioned previously in section 1.3, the line search strategy is an integral part of the global optimization routine. The basic structure of the line search algorithm is illustrated in Figure 3.3. As may be seen, when the strict line search termination criterion is employed (ICUB=0), an additional situation is introduced wherein cubic interpolation is used to reduce the size of the search bracket. The primary advantage of this strict termination criterion is that it permits exact line searches to be carried out in the limit as σ approaches zero. However, in most practical problems inexact line searches (say $\sigma \simeq 0.7\text{-}0.9$) normally result in a lesser total number of objective function evaluations. Hence, provided that σ is greater than about 0.7, ICUB=0 and ICUB=1 analyses will generally yield equally efficient results.

48
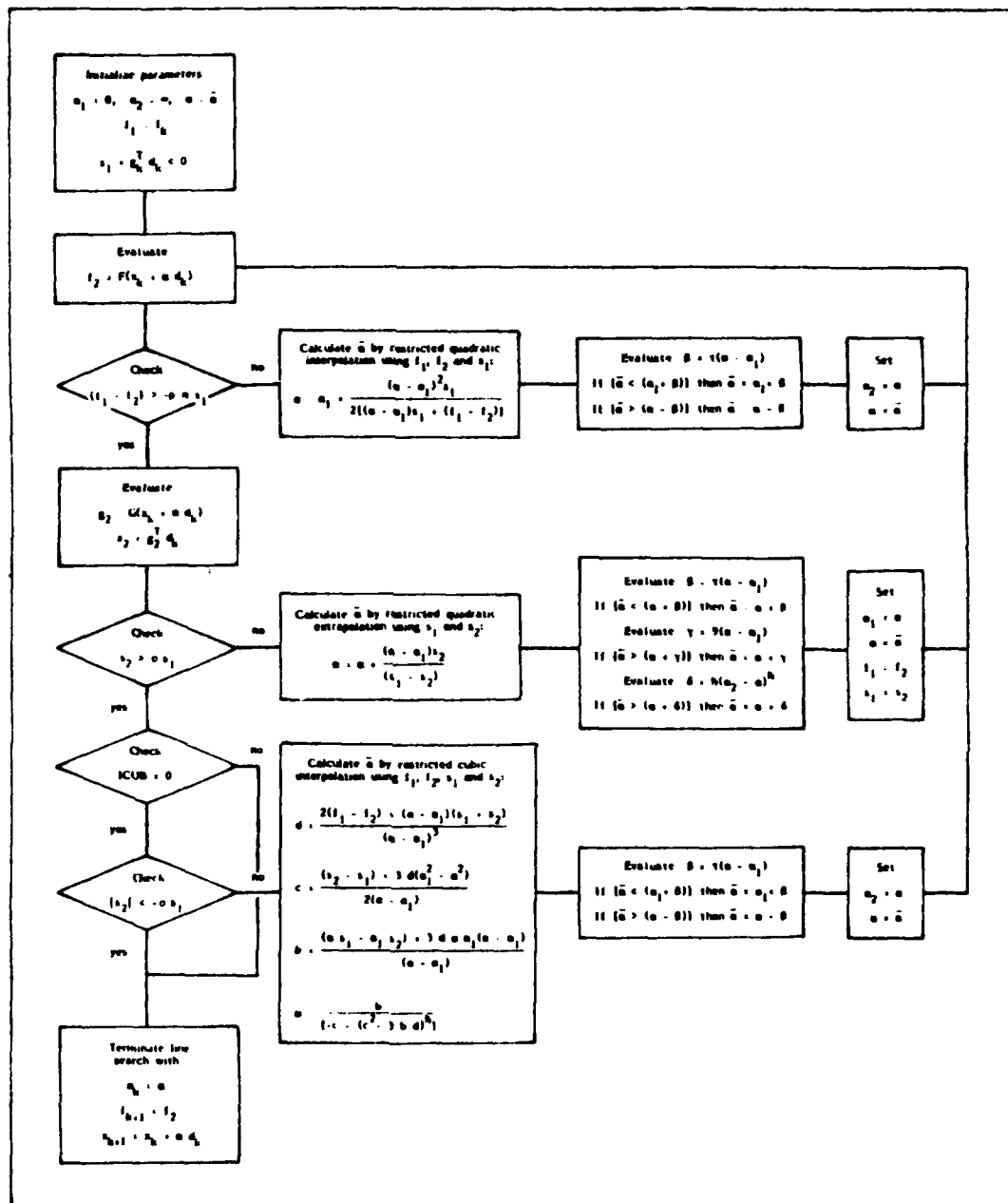
Figure 3.3: Structure Of The Line-Search Component
Of The Minimization Algorithm

The constants C1, C2 and C3 represent the line search parameters σ, ρ and τ, respectively. As indicated in Figure 3.3, the parameters σ and ρ control the degree of accuracy to which a particular line search is carried out; observe that as σ increases and ρ decreases, the line search becomes less and less exact. The parameter τ prevents evaluations from being made at points which are trivially close to the boundaries of the current search bracket. By default, σ = 0.90, ρ = 0.0001 and τ = 0.10.

Normally, the line search terminates because the exit criterion shown in Figure 3.3 has been satisfied. However, near the solution (that is, the minimum) round-off errors may become significant, and it is sometimes more efficient and reliable to use parameter values as the exit criterion. Thus, in the code, the line search is terminated whenever the predicted location of the optimum is no longer changing by a significant amount. Mathematically, this additional exit criterion is given by:

$$\text{EPXL} > \left| \frac{x^i_{j-1} - x^i_j}{x^i_{j-1}} \right| \qquad (i = 1, \text{NDIM})$$

where x^i_{j-1} and x^i_j represent the values of parameter x^i at two consecutive objective function evaluations. By default, EPXL=0.001.

There are a variety of criteria that may be used to test for global convergence and which may therefore serve as a basis for terminating the optimization search. Three of the most commonly used and effective criteria are:

i) based on parameter values (ICON=1),

$$\epsilon^i > | x^i_{k-1} - x^i_k |$$

ii) based on function values (ICON=2),

$$EPFA > (f_{k-1} - f_k)$$

iii) based on gradient values (ICON=3),

$$EPSA > |-g_k^T H_k g_k|$$

where the subscripts k-1 and k refer to the value of the quantity at the completion of the k-1th and kth line search, respectively.

The use of criterion (i) ensures that the location of the optimum has been determined to within the specified accuracy, but does not guarantee that the smallest function value has been found. The use of criterion (ii) requires the user to identify on input the maximum absolute error that can be tolerated with respect to the magnitude of the objective function at the optimum. The use of criterion (iii) may perhaps provide the best results; when the gradient is small in the next computed search direction, any additional searches in that direction should yield little or no reduction in the value of the objective function.

Regardless of which of the above three global termination criteria is used, a fail-safe exit condition is also incorporated, to prevent the algorithm from continuing when no substantial progress is being made. If at some point in the search

$$EPFL > (f_{k-1} - f_k)$$

and this condition continues to be satisfied for the next NFLX objective function evaluations, the search is terminated, regardless of ICON. This fail-safe exit condition was introduced to prevent unnecessary function evaluations from being made when either parameter values (ICON=1) or gradient values (ICON=3) are used as the convergence criterion, but where this criterion is, for one reason or another, too strict to ever be invoked. If function values are used as the convergence criterion (ICON=2), the user should set EPFA=EPFL.

51

The codes IHES and ISCA (and the constant SFUN of input section X) are used to select options which may improve the conditioning of the problem and therefore the efficiency of the search. If IHES=0, the initial inverse Hessian will be scaled prior to the first update. This scaling procedure was originally suggested by Shanno and Phua (1978) as a means for reducing the cancellation errors which may occur when the terms in the updating formula (see section 1.3) are of significantly different magnitudes. However, the results of the procedure have, in practice, been rather mixed (see Shanno and Phua (1978)). For small to medium sized problems (say NDIM < 10 to 15), such as those normally encountered in model calibration, it is probably best to set IHES=0, and thereby prevent scaling. For larger problems, it may be possible to improve the algorithm's performance by setting IHES=1.

When the typical values of the constitutive parameters are of vastly different magnitudes (such as a shear modulus of G=2500 psi, versus a projection point of c=0.25), the Quasi-Newton strategy may perform quite poorly as a result of badly scaled x, g and δ vectors (see section 1.3). It is often possible, though, to improve the routine's performance by making the variable transformation

$$y = Dx$$

and minimize, instead, $f[x(y)] = f(y)$, where D is a diagonal matrix consisting of NDIM scaling factors. However, since the program already employs a trigonometric variable transformation to enable the imposition of upper and lower bounds (see section 1.3), which generally results in a very well scaled problem, no additional scaling should normally be required (ISCA=0).

Finally, it is sometimes possible to improve the algorithm's performance by minimizing a scalar multiple of the objective function (SFUN • f), rather than the function itself. This phenomenon again appears to be due entirely to

the presence of round-off errors. However, since the optimal value of the scalar SFUN is extremely dependent on the specific problem being considered, there is normally no reason to initially invoke the function scaling option (and thus, by default, SFUN=1.0).

3.6) Input Section X

As mentioned previously in section 1.3, gradients are evaluated by means of finite differences, in accordance with the strategy originally suggested by Stewart (1967). To enable this strategy to be efficiently carried out, certain problem and machine dependent tolerance information must be supplied.

On a finite word length machine, a given property value x_i cannot be represented with infinite precision, but, rather, must be stored as some truncated approximation x_i^*, where:

$$x_i^* = (1 + \epsilon_1)x_i \qquad \text{and} \qquad \epsilon_1 < \text{ETAX}$$

Similarly, instead of providing a perfectly accurate value of the objective function f, the algorithm can only yield an approximation f^*, where:

$$f^* = (1 + \epsilon_2)f \qquad \text{and} \qquad \epsilon_2 < \text{ETAF}$$

The user must further specifiy the maximum truncation error that can be tolerated in the forward difference representation of derivatives, FDER. The constant FDER thus permits the code to identify when it is necessary to replace the forward difference formula with the more expensive central difference representation. Finally, in order to prevent the computed differencing intervals from becoming too small to yield a representative gradient, a lower limit on the size of the intervals must also be specified by means of the constant DMIN. Since the interval size check is made after the trigonometric variable transforma-

53

tion discussed in section 1.3 has been performed, this single bound DMIN can be applied to all components of the gradient vector and still produce the desired effect. By default, $ETAX=10^{-8}$, $ETAF=10^{-4}$, $FDER=10^{-4}$ and $DMIN=0.005$.

3.7) Input Sections XI Through XVI

The ordering of the specifications in input sections XI through XVI should be consistent with that used in input section II to identify the unknown constitutive parameters. For example, if a four-dimensional (NDIM=4) analysis was performed to establish the optimal values of R_c, A_c, c and h_c (having property numbers of 9, 10, 15 and 16, respectively, as shown in Table 2.1), and if the specification of input section II was made in the order:

$$KOPT (1) = 9$$
$$KOPT (2) = 10$$
$$KOPT (3) = 15$$
$$KOPT (4) = 16$$

then the initial estimate for R_c should occupy the first position (columns 1-10) of input section XI, the lower bound for c should occupy the third position (columns 21-30) of input section XII, and so forth. For convenience, typical starting estimates and lower and upper bounds are listed in Table 2.3.

The initial differencing intervals of input section XIV should be large enough to yield an appreciable change in the objective function, but not so large that the local geometry of the function is misrepresented. As a rule of thumb, the size of the initial differencing intervals should be about 5-10% as great as either (i) the initial estimate for the corresponding parameter, or (ii) the difference between the specified upper and lower bounds. For convenience, suggested interval sizes are given in Table 2.3.

54

3.8)   Input Section XVII

By default, the initial inverse Hessian matrix is set to H = I (see section 1.3). However, if the objective function's curvature at the starting location is known, an improved initial estimate for H could be specified by setting NHES > 0 in section VIII and entering the necessary second derivative information in section XVII. Since the initial inverse Hessian must be both symmetric and positive-definite, it is only necessary to enter the non-zero components on the diagonal and in the upper (or lower) triangular block.

3.9)   Input Section XVIII

Finally, if either R_c or R_e (or both) are among the parameters whose optimal values are being sought, improved initial estimates can be acquired by setting NSLN $\neq$ 0 in section VIII and entering the necessary data in section XVIII. These improved estimates are generated internally, by identifying the roots of the equation which defines the undrained stress path of a normally consolidated soil:

$$F(R) = 0 = 2(\alpha^\beta - \alpha^{2\beta}) + (\alpha^{2\beta} - 2)R + 2R^2 - R^3$$

where:

$$\alpha = \frac{p_f'}{p_o'}$$

$$\beta = \frac{-\lambda}{\lambda - \kappa}$$

and where λ and κ represent the slopes of the material's isotropic consolidation and swell/recompression curves in e − ln p′ space, and p_o' and p_f' denote the initial and critical (or "failure") values of the mean normal effective stress p′, as indicated in Figure 3.4.

55
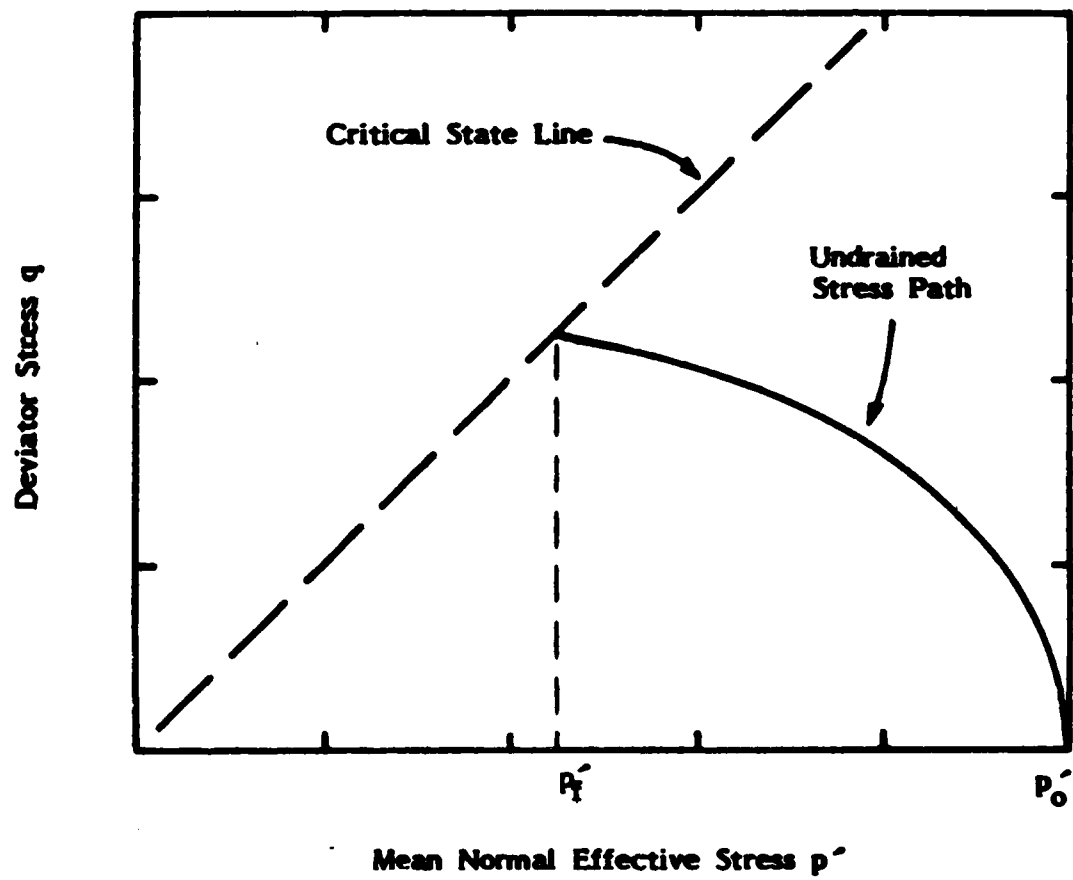
Figure 3.4: The Undrained Stress Path For A Typical Normally Consolidated Soil Specimen

The values of R which satisfy the above cubic equation are obtained through the use of a modified-Newton root finding scheme. The structure of the search algorithm is as follows:

i) estimate the location of the root R

ii) compute $u(R) = \dfrac{f(R)}{f'(R)}$

iii) compute $u'(R) = 1 - \dfrac{f(R)f''(R)}{[f'(R)]^2}$

iv) compute $R^{n+1} = R^n - \dfrac{u(R)}{u'(R)} = R^n + \delta^n$

v) if $\epsilon > |\delta^n|$ then stop, otherwise repeat steps (ii)-(iv)

Although a cubic equation, in general, possesses three roots, the equation presented above has normally been found to have only a single real root in the approximate range $1.5 < R < 3.5$ (where the final value of R should nearly always lie). Hence, the internally computed estimate for R should typically lead to a rather good correlation between the observed and predicted responses associated with a normally consolidated soil. If improved estimates for both R_c and R_e are required (NSLN = 2), the specifications pertaining to R_c must be entered first.

57

IV. MODIFYING THE CODE

The program MODCAL incorporates 29 separate subroutines, each of which performs a limited and specific task. This modular form gives the code maximum generality and extreme versatility. For example, if, in the future, another optimization strategy is found to be more effective than the Quasi-Newton procedure presently employed, the associated subroutines may be readily replaced, without having to additionally modify the other components of the code. Or, if it was desired to use boundary-value measurements to calibrate the material model, rather than homogeneous test results, subroutine EVAL could easily be replaced with an appropriate multi-element finite element program (such as the nonlinear two-dimensional code NTD developed by Herrmann et al (1982b)). Finally, if it was desired to adopt the present automated calibration strategy to another constitutive formulation, subroutines CLAY and BOUNDS could rapidly be replaced with the appropriate material model. The precise procedure for linking a new materials subroutine to the driving subroutine EVAL is discussed in detail by Herrmann et al (1980, 1981).

The code's modular structure also enables it to be readily implemented on limited core memory minicomputers, such as the LSI-11/23. The presence of many small subroutines facilitates the use of "overlays", wherein the various subroutines are swapped into and out of core as they are needed. The use of overlays therefore often makes it possible to accommodate large programs which would otherwise exceed the core memory capacity of a small machine. Alternatively, if the code is to be used only in a limited form (such as for performing model calibration only, but without plotting), the unnecessary subroutines may simply be removed, thus reducing the size of the program.

In order to facilitate the use of these various options, and to enable the user to more clearly identify where particular modifications may be required,

58

a listing of the 29 subroutines and their respective functions will now be presented:

Subroutine	Function
1. MODCAL:	the main driving program for the Bounding Surface soil plasticity model calibration and prediction code
2. DATAIN:	directs the reading of the required input information and control specifications
3. FNDMAX:	establishes the maxima of the specified experimental observations
4. EVAL:	directs the single-element, incremental-iterative finite element analysis which generates the set of model predictions for the specified homogeneous test conditions
5. AITKEN:	computes the appropriate Aitken's acceleration factors
6. PLTCHK:	checks whether a given experimentally observed or theoretically predicted response relation is to be plotted
7. JSDPLT:	directs the plotting of the experimental observations and model predictions
8. LSHIFT:	adjusts the minimum and maximum axes values, as directed by JSDPLT
9. BORDER:	produces the plot headings and axes lables, as directed by JSDPLT
10. RSDUAL:	directs the evaluation of the weighted residuals and forms the governing objective function

Subroutine	Function
11. **EUCLID:**	computes the minimum Euclidean distance between the experimental observation and the prediction curve
12. **VRTICL:**	computes the minimum vertical distance between the experimental observation and the prediction curve
13. **BSMOPT:**	directs the global search component of the model calibration algorithm
14. **CALDAT:**	directs the reading of all additional specifications needed for model calibration
15. **NEWTON:**	computes improved starting estimates for the surface shape parameters R_c and/or R_e
16. **SEARCH:**	directs the line search component of the minimization algorithm
17. **BFGSUP:**	computes the inverse Hessian matrix H, in accordance with the BFGS (Broyden-Fletcher-Goldfarb-Shanno) updating formula
18. **CONCHK:**	checks whether the optimization algorithm has converged to the minimum
19. **PRNOUT:**	directs the printing of the minimization search results
20. **DTPROD:**	computes the scalar (dot) product of two vectors $v^T v$
21. **VVPROD:**	computes the product of the vector-vector operation vv^T
22. **MVPROD:**	computes the product of the matrix-vector operation Mv
23. **MMPROD:**	computes the product of the matrix-matrix operation MM

Subroutine	Function
24. FFUN:	computes the value of the objective function
25. GFUN:	computes the gradient of the objective function
26. OPEN:	opens the required input and output files
27. EXIT:	closes the required input and output files
28. CLAY:	contains the numerical implementation of the governing constitutive equations, and thus provides the appropriate theoretical material response to the given stress and/or strain increment, as supplied by EVAL
29. BOUNDS:	evaluates the relationship of the stress state to the bounding surface, as required by CLAY.

V. EXPANDING THE CAPABILITIES OF THE CODE

The program MODCAL employs a number of 1-, 2- and 3-dimensional arrays. In order to minimize storage requirements, or to enable the code to accommodate larger problems, it may at times be necessary to adjust the dimensions of the size-dependent arrays. The list which follows describes the various size-dependent arrays, dimensioned in terms of a few key constants:

in COMMON BLKA:

KOPT (NDIM)

in COMMON BLKB:

W1 (NTST)

W2 (NTST, NPLT)

W3 (NTST, NPLT, NEXP)

in COMMON BLKC:

NSEG (NTST)

PRP2 (NTST, 4)

LTYP (NTST, NSEG, 7)

VALU (NTST, NSEG, 7)

in COMMON BLKD:

NPLT (NTST)

KPLT (NTST, NPLT)

NEXP (NTST, NPLT)

XV (NTST, NPLT, NEXP)

YV (NTST, NPLT, NEXP)

PINC (NINC + 1, NVAR)

in COMMON BLKE:

XMXR (NRLN)

YMXR (NRLN)

XMXP (NTST, NPLT)

YMXP (NTST, NPLT)

in COMMON BLK3:

X1 (NDIM)

X2 (NDIM)

in COMMON BLK4:

XE (NDIM)

in COMMON BLK5:

D1 (NDIM)

D2 (NDIM)

DG (NDIM)

DX (NDIM)

G1 (NDIM)

G2 (NDIM)

H1 (NDIM, NDIM)

H2 (NDIM, NDIM)

in COMMON BLK7:

SF (NDIM)

XL (NDIM)

XU (NDIM)

in MAIN:

XV (NDIM)

in subroutine EVAL:

XOPT (NDIM)

in subroutine PLTCHK:

IPLT (NRLN)

in subroutine JSDPLT:

      ICEX (NEXP)

      ICEY (NEXP)

      ICPX (NINC + 1)

      ICPY (NINC + 1)

in subroutines BSMOPT, FFUN and GFUN:

      XX (NDIM)

in subroutine SEARCH:

      DM (NDIM)

      XM (NDIM)

in subroutine BFGSUP:

      SV (NDIM)

      SM (NDIM, NDIM)

in subroutine CONCHK:

      XV (NDIM)

in subroutine PRNOUT:

      XS (NDIM)

in subroutines DTPROD, VVPROD, MVPROD and MMPROD:

      V1 (NDIM)

      V2 (NDIM)

      V3 (NDIM, NDIM)

      V4 (NDIM, NDIM)

where, by definition:

NDIM $\geq$ number of parameters whose optimal values are to be established by optimization;

NTST $\geq$ number of distinct experimental tests;

NPLT $\geq$ maximum number of response relations associated with a single experimental test;

64

NEXP $\geqslant$ maximum number of experimental observations associated with a single response relation;

NSEG $\geqslant$ maximum number of history segments associated with a single prescribed loading history;

NINC $\geqslant$ maximum number of increments into which a single prescribed loading history has been subdivided;

NVAR $\geqslant$ number of different response parameters (such as q, p´, u, ϵ_1 and ϵ_v) used to define the results of the various experimental tests; and,

NRLN $\geqslant$ number of different response relations (such as q vs p´, q vs ϵ_1, p´ vs ϵ_1, u vs ϵ_1 and ϵ_v vs ϵ_1) used to define the results of the various experimental tests.

The program is currently set up to accommodate five different response relations (NRLN = 5), which, as a group, are fully defined by means of five distinct response parameters (NVAR = 5). If, for some reason, it becomes necessary to use relations and/or parameters that are different from those listed above to characterize the results of a given experimental test, minor changes must be made to the coding which handles the storage and retrieval of the model predictions. These coding changes are quite straightforward, and are restricted only to subroutines EVAL, PLTCHK and RSDUAL. The remaining dimensions are currently set at NDIM = 17, NTST = 6, NPLT = 3, NEXP = 20, NSEG = 4 and NINC = 60.

65

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XVII. EXAMPLE I: MODEL CALIBRATION

To provide a first example of the code's capabilities, MODCAL has been used to assist in the calibration of the Bounding Surface plasticity model for a particular cohesive soil. As may be seen from the input and output files which follow, the program was run to establish the optimal values of the five model parameters G, R_c, A_c, c and h_c for the case of the laboratory prepared Kaolin employed by Jafroudi (1983) (see also Herrmann et al (1981b)). The specified calibration data base includes the results of conventional undrained triaxial compression tests (in the form of the experimentally observed q vs ϵ_1, p´ vs ϵ_1 and u vs ϵ_1 responses) performed on specimens at overconsolidation ratios of OCR = 1, 2 and 6. All experimental observations are weighted equally, and the objective function is formed by using the absolute-Euclidean measure of error (KNRM=0 and KRSL=0), together with the values PLIM = 0.01 and TLIM = 0.20.

```
EXAMPLE I: MODEL CALIBRATION.  JAFROUDI (1983) KAOLIN -- OCR=1,2,6.
    3    1    0    0    0    0    5    5    9   10   14   16
      0.130      0.018      1.180      0.737   5000.00 1000000.0    101.35    101.35
      2.400      0.100      0.050      1.000      1.000      0.400      1.000
      0.500      0.500      1.000      1.000
    1          0.68     392.20     392.20       1.00
    2          0.69     392.20     196.10       2.00
    3          0.72     392.20      65.37       6.00
    0   10    0          1.00       0.01       0.01       0.20
    1    2
0 392.2  0  392.2 1  0.02 1       1          1                    20      1.00
0 392.2  0  392.2 1  0.12 1       1          1                    20      1.00
    2    2
0 196.1  0  196.1 1  0.02 1       1          1                    20      1.00
0 196.1  0  196.1 1  0.12 1       1          1                    20      1.00
    3    2
0  65.4  0   65.4 1  0.02 1       1          1                    20      1.00
0  65.4  0   65.4 1  0.12 1       1          1                    20      1.00
    1    3
    2   15              3   15              4   15
      0.0       65.0      105.0      155.0      176.0      187.0      202.0      208.0
    214.0      216.0      217.0      217.0      215.0      212.0      206.0
      0.00       0.12       0.25       0.50       0.75       1.00       1.50       2.00
      3.00       4.00       5.00       6.00       8.00      10.00      12.00
    392.2      383.9      362.2      329.9      304.9      284.5      256.5      236.5
    216.5      204.2      196.5      191.5      184.9      179.9      174.9
      0.00       0.12       0.25       0.50       0.75       1.00       1.50       2.00
      3.00       4.00       5.00       6.00       8.00      10.00      12.00
      0.0       30.0       65.0      114.0      146.0      170.0      203.0      225.0
    247.0      260.0      268.0      273.0      279.0      283.0      286.0
      0.00       0.12       0.25       0.50       0.75       1.00       1.50       2.00
      3.00       4.00       5.00       6.00       8.00      10.00      12.00
    2    3
    2   15              3   15              4   15
      0.0       50.0       70.0      135.0      155.0      169.0      180.0      187.0
    195.0      199.0      202.0      202.0      202.0      200.0      198.0
      0.00       0.12       0.25       0.50       0.75       1.00       1.50       2.00
      3.00       4.00       5.00       6.00       8.00      10.00      12.00
    196.1      197.8      199.0      206.1      203.8      201.4      194.1      189.4
    181.1      175.4      171.4      169.4      165.4      161.1      160.1
      0.00       0.12       0.25       0.50       0.75       1.00       1.50       2.00
      3.00       4.00       5.00       6.00       8.00      10.00      12.00
      0.0       15.0       23.0       35.0       44.0       51.0       62.0       69.0
     80.0       87.0       92.0       94.0       98.0      100.0      102.0
      0.00       0.12       0.25       0.50       0.75       1.00       1.50       2.00
      3.00       4.00       5.00       6.00       8.00      10.00      12.00
    3    3
    2   15              3   15              4   15
      0.0       25.0       52.0       80.0       96.0      109.0      125.0      135.0
    148.0      155.0      159.0      161.0      163.0      162.0      159.0
      0.00       0.12       0.25       0.50       0.75       1.00       1.50       2.00
      3.00       4.00       5.00       6.00       8.00      10.00      12.00
     65.4       71.7       79.7       90.0       97.4      102.7      110.0      116.4
    122.7      128.0      130.4      131.0      131.7      131.4      130.4
```

```
   0.00      0.12      0.25      0.50      0.75      1.00      1.50      2.00
   3.00      4.00      5.00      6.00      8.00     10.00     12.00
   0.0       2.0       3.0       2.0       0.0      -1.0      -3.0      -6.0
  -8.0     -11.0     -12.0     -12.0     -12.0     -12.0     -12.0
   0.00      0.12      0.25      0.50      0.75      1.00      1.50      2.00
   3.00      4.00      5.00      6.00      8.00     10.00     12.00
 5  400     2   3     0    0     0    0     1
1.00E-02  1.00E-08  9.00E-01  1.00E-04  1.00E-01  1.00E-03  1.00E-04  100
1.00E-04  1.00E-08  5.00E-03  1.00E-04  1.00E+00
5000.00    2.40      0.10      0.40      0.50
2500.00    2.00      0.03      0.00      0.05
10000.00   3.00      0.20      0.80      2.00
500.00     0.05      0.01      0.05      0.05
0.130      0.018    392.20    175.00     2.40      1
```

70

EXAMPLE I: MODEL CALIBRATION. JAFROUDI (1983) KAOLIN -- OCR=1,2,6.

INPUT DATA:
CONTROL CODES:

```
        JOPT  =  1
        JRUN  =  0
        JPLT  =  0
        NTST  =  3
        KRSL  =  0
        KNRM  =  0
        POWR  =  1.00E+00
        PLIM  =  1.00E-02
        TLIM  =  2.00E-01
        NOPT  =  5
        KOPT  =  5     9    10    14    16
```


INPUT DATA:
MATERIAL PROPERTIES:

```
        LAMBDA                 = 1.300E-01
        KAPPA                  = 1.800E-02
        MUC (CSL SLOPE)        = 1.180E+00
        MUE/MUC                = 7.370E-01
        SHEAR MODULUS  G       = 5.000E+03
        CBULK MODULUS  GAMMA   = 1.000E+06
        TRANSITIONAL STRESS  PL = 1.013E+02
        ATMOSPHERIC PRESSURE PA = 1.013E+02
```


MODEL CONSTANTS:

```
        RC     = 2.400E+00
        AC     = 1.000E-01
        T      = 5.000E-02
        RE/RC  = 1.000E+00
        AE/AC  = 1.000E+00
        PROJECTION POINT C = 4.000E-01
        ELASTIC  NUCLEUS S = 1.000E+00
        HC     = 5.000E-01
        MC     = 5.000E-01
        HE/HC  = 1.000E+00
        ME/MC  = 1.000E+00
```

71

INITIAL STATE PROPERTIES:

TEST	OCR	VOID-R	P-PRECON	P-CONFIN
1	1.00	0.68	3.922E+02	3.922E+02
2	2.00	0.69	3.922E+02	1.961E+02
3	6.00	0.72	3.922E+02	6.537E+01

INPUT DATA:
ITERATION INFORMATION AND ANALYTICAL OPTIONS:

      MAX # OF ITERATIONS = 10
      MAX RELATIVE ERRORS = 1.000E-02
      ACCELERATION FACTOR LIMITS = 1.000E+00

  **** UNDRAINED CONDITIONS ****

  **** ENGINEERING STRESSES AND STRAINS ASSUMED ****

  **** REFORMULATED NEARLY INCOMPRESSIBLE ANALYSIS ****

INPUT DATA:
SPECIFIED LOADING HISTORIES:

TEST	SEG-#	#-INC	S-RATIO	SS-CODE	SIG/EPS-X	SIG/EPS-Y	SIG/EPS-Z	TAU/GAM-XY	TAU/GAM-XY	TAU/GAM-YZ
1	1	20	1.00	001111	3.922E+02	3.922E+02	2.000E-02	0.000E-01	0.000E-01	0.000E-01
1	2	20	1.00	001111	3.922E+02	3.922E+02	1.200E-01	0.000E-01	0.000E-01	0.000E-01
2	1	20	1.00	001111	1.961E+02	1.961E+02	2.000E-02	0.000E-01	0.000E-01	0.000E-01
2	2	20	1.00	001111	1.961E+02	1.961E+02	1.200E-01	0.000E-01	0.000E-01	0.000E-01
3	1	20	1.00	001111	6.540E+01	6.540E+01	2.000E-02	0.000E-01	0.000E-01	0.000E-01
3	2	20	1.00	001111	6.540E+01	6.540E+01	1.200E-01	0.000E-01	0.000E-01	0.000E-01

INPUT DATA:
SPECIFIED EXPERIMENTAL OBSERVATIONS AND WEIGHTING FACTORS:

TEST	PLOT	KIND	WF-TEST	WF-PLOT	#-PTS	YMAX(PLOT)	XMAX(PLOT)	YMAX(KIND)	XMAX(KIND)
1	1	2	1.000	1.000	15	2.170E+02	1.200E+01	2.170E+02	1.200E+01
1	2	3	1.000	1.000	15	3.922E+02	1.200E+01	3.922E+02	1.200E+01
1	3	4	1.000	1.000	15	2.860E+02	1.200E+01	2.860E+02	1.200E+01
2	1	2	1.000	1.000	15	2.020E+02	1.200E+01	2.170E+02	1.200E+01
2	2	3	1.000	1.000	15	2.061E+02	1.200E+01	3.922E+02	1.200E+01
2	3	4	1.000	1.000	15	1.020E+02	1.200E+01	2.860E+02	1.200E+01
3	1	2	1.000	1.000	15	1.630E+02	1.200E+01	2.170E+02	1.200E+01
3	2	3	1.000	1.000	15	1.317E+02	1.200E+01	3.922E+02	1.200E+01
3	3	4	1.000	1.000	15	1.200E+01	1.200E+01	2.860E+02	1.200E+01

```
INPUT DATA:
CALIBRATION CONTROL CODES:
----------------------------

                NUMBER OF DIMENSIONS                   =   5
                MAXIMUM NUMBER OF FUNCTION EVALUATIONS  = 400
                OUTPUT DATA PRINT CODE                  =   2
                CONVERGENCE CRITERION CODE              =   3
                CUBIC INTERPOLATION OPTION CODE         =   0
                HESSIAN CONDITIONING OPTION CODE        =   0
                # OF NON-ZERO HESSIAN VALUES TO BE READ =   0
                PARAMETER SCALING OPTION CODE           =   0
                # OF IMPROVED STARTING ESTIMATES REQUIRED =   1
                ABSOLUTE CONVERGENCE CRITERION FOR F(X) = 1.000E-02
                ABSOLUTE CONVERGENCE CRITERION FOR S(X) = 1.000E-08
                VALUE OF THE LINE-SEARCH PARAMETER, SIGMA = 9.000E-01
                VALUE OF THE BRACKET-CHECK PARAMETER, ROA = 1.000E-04
                VALUE OF THE BRACKET-CHECK PARAMETER, TAU = 1.000E-01
                VALUE OF THE LINE SEARCH EXIT CRITERION  = 1.000E-03
                VALUE OF THE FAIL-SAFE GLOBAL CRITERION  = 1.000E-04
                # OF FN-EVALUATIONS BEFORE FAIL-SAFE EXIT = 100
                ERROR BOUND: MACHINE REPRESENTATION OF XX = 1.000E-08
                ERROR BOUND: MACHINE REPRESENTATION OF FX = 1.000E-04
                MINIMUM VALUE OF THE DIFFERENCE INTERVALS = 5.000E-03
                MAXIMUM ERROR IN FWD-DIFFERENCE CALCULATN = 1.000E-04
                SCALING FACTOR FOR THE OBJECTIVE FUNCTION = 1.000E+00


INPUT DATA:
INITIAL VALUES OF THE FUNCTION VARIABLES X1,X2,...,XN:
-------------------------------------------------------------

        5.00E+03  2.55E+00  1.00E-01  4.00E-01  5.00E-01

INPUT DATA:
LOWER AND UPPER BOUNDS ON THE FUNCTION VARIABLES X1,X2,...,XN:
-------------------------------------------------------------

        2.50E+03  2.00E+00  3.00E-02  0.00E-01  5.00E-02
        1.00E+04  3.00E+00  2.00E-01  8.00E-01  2.00E+00

INPUT DATA:
INITIAL VALUES OF THE DIFFERENCING INTERVALS D1,D2,...,DN:
-------------------------------------------------------------

        5.00E+02  5.00E-02  1.00E-02  5.00E-02  5.00E-02
```

OUTPUT DATA:
THE RESULTS OF THE OPTIMIZATION ROUTINE FOLLOW:

#-I	#-F	#-G	F1	F2	S1	S2	X1	X2	X3	X4	X5
0	6	1	0.000E-01	1.209E-01	0.000E-01	-1.162E-01	5.000E+03	2.551E+00	1.000E-01	4.000E-01	5.000E-01
1	17	2	1.209E-01	1.140E-01	-1.162E-01	3.986E-02	5.918E+03	2.520E+00	7.425E-02	4.984E-01	1.790E-01
2	28	3	1.140E-01	1.057E-01	-2.221E-02	-3.672E-03	6.006E+03	2.534E+00	6.624E-02	4.407E-01	2.633E-01
3	44	4	1.057E-01	1.057E-01	-8.100E-03	-8.479E-03	6.006E+03	2.534E+00	6.624E-02	4.407E-01	2.633E-01
4	64	5	1.057E-01	1.057E-01	-7.830E-01	9.880E-01	6.006E+03	2.534E+00	6.624E-02	4.407E-01	2.633E-01
6	112	8	1.020E-01	1.020E-01	-1.939E+00	-6.981E-01	6.475E+03	2.574E+00	6.393E-02	4.441E-01	3.008E-01
7	126	9	1.020E-01	1.020E-01	-5.955E-04	3.318E-03	6.476E+03	2.574E+00	6.393E-02	4.441E-01	3.008E-01
8	137	10	1.020E-01	1.020E-01	-1.210E-06	2.506E-05	6.474E+03	2.574E+00	6.392E-02	4.441E-01	3.009E-01
9	148	11	1.020E-01	1.020E-01	-2.323E-05	7.391E-06	6.475E+03	2.574E+00	6.393E-02	4.441E-01	3.008E-01

***** THE INVERSE HESSIAN HAS BEEN RESET TO THE IDENTITY MATRIX *****

10	171	12	1.020E-01	1.008E-01	-9.859E-02	2.457E-02	6.519E+03	2.571E+00	6.243E-02	4.587E-01	2.607E-01
11	184	13	1.008E-01	1.008E-01	-3.280E-02	-5.474E-03	6.546E+03	2.570E+00	5.712E-02	4.656E-01	2.556E-01
12	199	14	1.008E-01	1.008E-01	-8.714E-04	-3.526E-04	6.545E+03	2.570E+00	5.711E-02	4.655E-01	2.555E-01
13	214	15	1.008E-01	1.008E-01	-9.221E-04	-1.240E-03	6.545E+03	2.570E+00	5.711E-02	4.656E-01	2.556E-01
14	227	16	1.008E-01	1.008E-01	-2.456E-04	-1.185E-04	6.545E+03	2.570E+00	5.711E-02	4.656E-01	2.556E-01
15	241	17	1.008E-01	1.008E-01	-2.144E-04	-3.188E-04	6.545E+03	2.570E+00	5.712E-02	4.656E-01	2.556E-01
16	267	19	1.007E-01	1.009E-01	-5.078E-04	-5.078E-04	6.542E+03	2.573E+00	5.732E-02	4.651E-01	2.576E-01

***** THE SEARCH HAS BEEN TERMINATED DUE TO INSUFFICIENT PROGRESS *****

XVIII. EXAMPLE II: MODEL PREDICTION

To provide a second example of the code's capabilities, MODCAL has been used to generate model predictions for a typical set of experimental test conditions. As may be seen from the input and output files which follow, predictions are required for three distinct loading histories. These three loading histories simulate the conventional undrained triaxial compression tests performed by Jafroudi (1983) on samples of Kaolin at overconsolidation ratios of OCR = 1, 2 and 6 (see also Herrmann et al (1981b)). For completeness, the relation plotting option has also been invoked (JPLT = 1), and in the nine plots which result, the symbol "#" represents an experimental observation, while the symbol "*" denotes a model prediction.

EXAMPLE II: MODEL PREDICTION. JAFROUDI (1983) KAOLIN -- OCR=1,2,6.

```
    3    0    1    1
    0.130      0.018     1.180     0.737     5900.0 1000000.0    101.35    101.35
    2.509      0.031     0.046     0.900     1.096      0.453     1.000
    0.621      0.500     1.377     1.000
    1          0.68    392.20    392.20      1.00
    2          0.69    392.20    196.10      2.00
    3          0.72    392.20     65.37      6.00
    0   10    0              1.00      0.01
    1    2
0  392.2  0  392.2  1   0.02  1          1         1                   20      1.00
0  392.2  0  392.2  1   0.12  1          1         1                   20      1.00
    2    2
0  196.1  0  196.1  1   0.02  1          1         1                   20      1.00
0  196.1  0  196.1  1   0.12  1          1         1                   20      1.00
    3    2
0   65.4  0   65.4  1   0.02  1          1         1                   20      1.00
0   65.4  0   65.4  1   0.12  1          1         1                   20      1.00
    1    3
    1   15              2   15              4   15
    0.0       65.0     105.0     155.0     176.0     187.0     202.0     208.0
  214.0      216.0     217.0     217.0     215.0     212.0     206.0
  392.2      383.9     362.2     329.9     304.9     284.5     256.5     236.5
  216.5      204.2     196.5     191.5     184.9     179.9     174.9
    0.0       65.0     105.0     155.0     176.0     187.0     202.0     208.0
  214.0      216.0     217.0     217.0     215.0     212.0     206.0
    0.00      0.12      0.25      0.50      0.75      1.00      1.50      2.00
    3.00      4.00      5.00      6.00      8.00     10.00     12.00
    0.0       30.0      65.0     114.0     146.0     170.0     203.0     225.0
  247.0      260.0     268.0     273.0     279.0     283.0     286.0
    0.00      0.12      0.25      0.50      0.75      1.00      1.50      2.00
    3.00      4.00      5.00      6.00      8.00     10.00     12.00
    2    3
    1   15              2   15              4   15
    0.0       50.0      70.0     135.0     155.0     169.0     180.0     187.0
  195.0      199.0     202.0     202.0     202.0     200.0     198.0
  196.1      197.8     199.0     206.1     203.8     201.4     194.1     189.4
  181.1      175.4     171.4     169.4     165.4     161.1     160.1
    0.0       50.0      70.0     135.0     155.0     169.0     180.0     187.0
  195.0      199.0     202.0     202.0     202.0     200.0     198.0
    0.00      0.12      0.25      0.50      0.75      1.00      1.50      2.00
    3.00      4.00      5.00      6.00      8.00     10.00     12.00
    0.0       15.0      23.0      35.0      44.0      51.0      62.0      69.0
   80.0       87.0      92.0      94.0      98.0     100.0     102.0
    0.00      0.12      0.25      0.50      0.75      1.00      1.50      2.00
    3.00      4.00      5.00      6.00      8.00     10.00     12.00
    3    3
    1   15              2   15              4   15
    0.0       25.0      52.0      80.0      96.0     109.0     125.0     135.0
  148.0      155.0     159.0     161.0     163.0     162.0     159.0
   65.4       71.7      79.7      90.0      97.4     102.7     110.0     116.4
  122.7      128.0     130.4     131.0     131.7     131.4     130.4
    0.0       25.0      52.0      80.0      96.0     109.0     125.0     135.0
  148.0      155.0     159.0     161.0     163.0     162.0     159.0
```

0.00	0.12	0.25	0.50	0.75	1.00	1.50	2.00
3.00	4.00	5.00	6.00	8.00	10.00	12.00	
0.0	2.0	3.0	2.0	0.0	-1.0	-3.0	-6.0
-8.0	-11.0	-12.0	-12.0	-12.0	-12.0	-12.0	
0.00	0.12	0.25	0.50	0.75	1.00	1.50	2.00
3.00	4.00	5.00	6.00	8.00	10.00	12.00	

EXAMPLE II: MODEL PREDICTION. JAFROUDI (1983) KAOLIN -- OCR=1,2,6.

INPUT DATA:
CONTROL CODES:

JOPT   = 0
JRUN   = 1
JFLT   = 1
NTST   = 3
KRSL   = 0
KNRM   = 0
POWR   = 1.00E+00
PLIM   = 1.00E-02
TLIM   = 1.00E-02
NOPT   = 0
KOPT   = 0

INPUT DATA:
MATERIAL PROPERTIES:

LAMBDA   = 1.300E-01
KAPPA   = 1.900E-02
MUC (CSL SLOPE)   = 1.180E+00
MUE/MUC   = 7.370E-01
SHEAR MODULUS G   = 5.900E+03
CBULK MODULUS GAMMA   = 1.000E+06
TRANSITIONAL STRESS PL   = 1.013E+02
ATMOSPHERIC PRESSURE PA   = 1.013E+02

MODEL CONSTANTS:

RC   = 2.509E+00
AC   = 3.100E-02
T   = 4.600E-02
RE/RC = 9.000E-01
AE/AC = 1.096E+00
PROJECTION POINT C = 4.530E-01
ELASTIC NUCLEUS S = 1.000E+00
HC   = 6.210E-01
MC   = 5.000E-01
HE/HC = 1.377E+00
MC/MC = 1.000E+00

79

INITIAL STATE PROPERTIES:

TEST	OCR	VOID-R	P-PRECON	P-CONFIN
1	1.00	0.68	3.922E+02	3.922E+02
2	2.00	0.69	3.922E+02	1.961E+02
3	6.00	0.72	3.922E+02	6.537E+01

INPUT DATA:
ITERATION INFORMATION AND ANALYTICAL OPTIONS:

MAX # OF ITERATIONS = 10
MAX RELATIVE ERRORS = 1.000E-02
ACCELERATION FACTOR LIMITS = 1.000E+00

UNDRAINED CONDITIONS

ENGINEERING STRESSES AND STRAINS ASSUMED

REFORMULATED NEARLY INCOMPRESSIBLE ANALYSIS

INPUT DATA:
SPECIFIED LOADING HISTORIES:

TEST	SEG-#	#-INC	S-RATIO	SS-CODE	SIG/EPS-X	SIG/EPS-Y	SIG/EPS-Z	TAU/GAM-XY	TAU/GAM-XY	TAU/GAM-YZ
1	1	20	1.00	001111	3.922E+02	3.922E+02	2.000E-02	0.000E-01	0.000E-01	0.000E-01
1	2	20	1.00	001111	3.922E+02	3.922E+02	1.200E-01	0.000E-01	0.000E-01	0.000E-01
2	1	20	1.00	001111	1.961E+02	1.961E+02	2.000E-02	0.000E-01	0.000E-01	0.000E-01
2	2	20	1.00	001111	1.961E+02	1.961E+02	1.200E-01	0.000E-01	0.000E-01	0.000E-01
3	1	20	1.00	001111	6.540E+01	6.540E+01	2.000E-02	0.000E-01	0.000E-01	0.000E-01
3	2	20	1.20	001111	6.540E+01	6.540E+01	1.200E-01	0.000E-01	0.000E-01	0.000E-01

80

OUTPUT DATA:
MODEL PREDICTIONS CORRESPONDING TO TEST DATA SET # 1 ARE AS FOLLOWS:

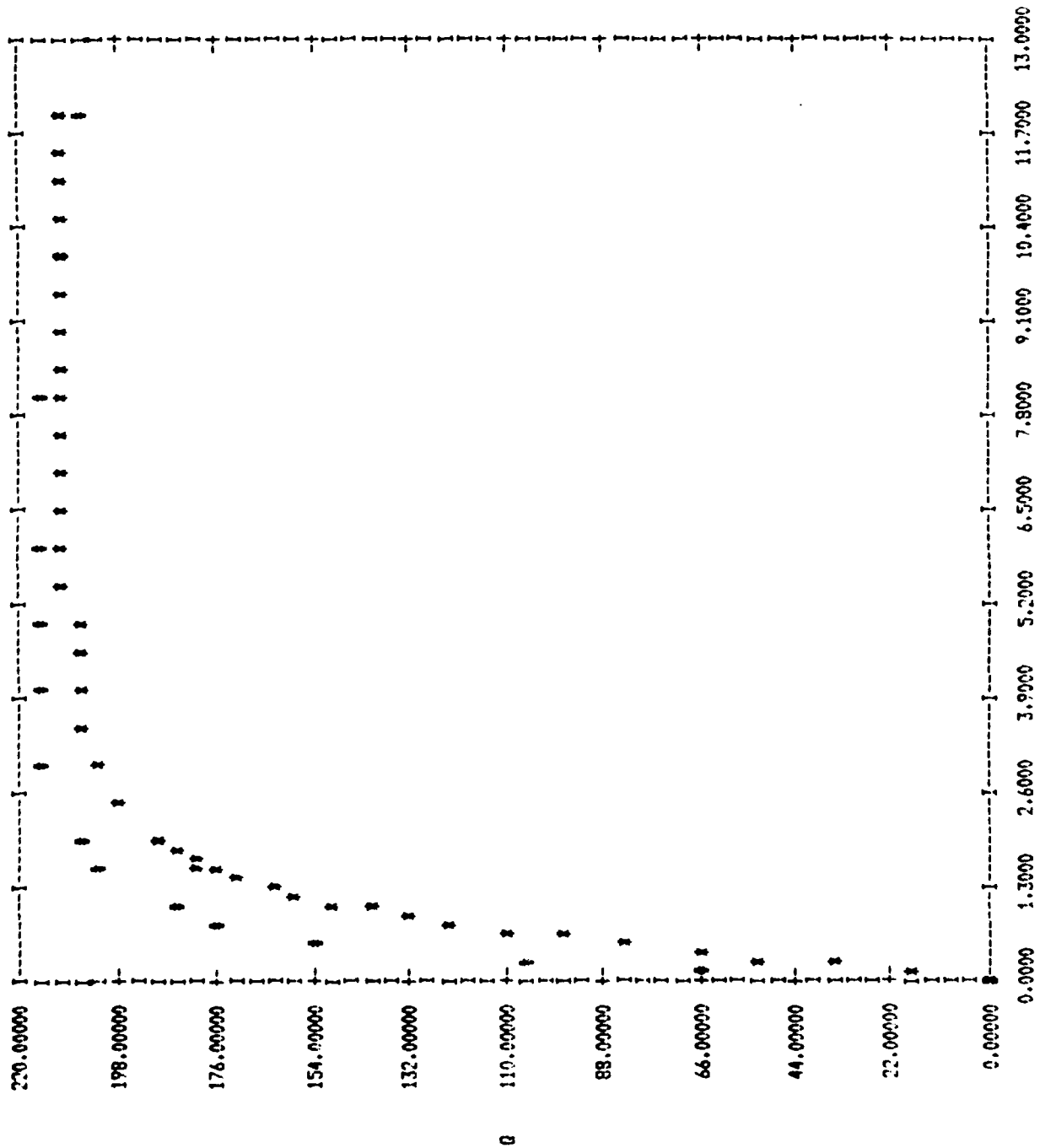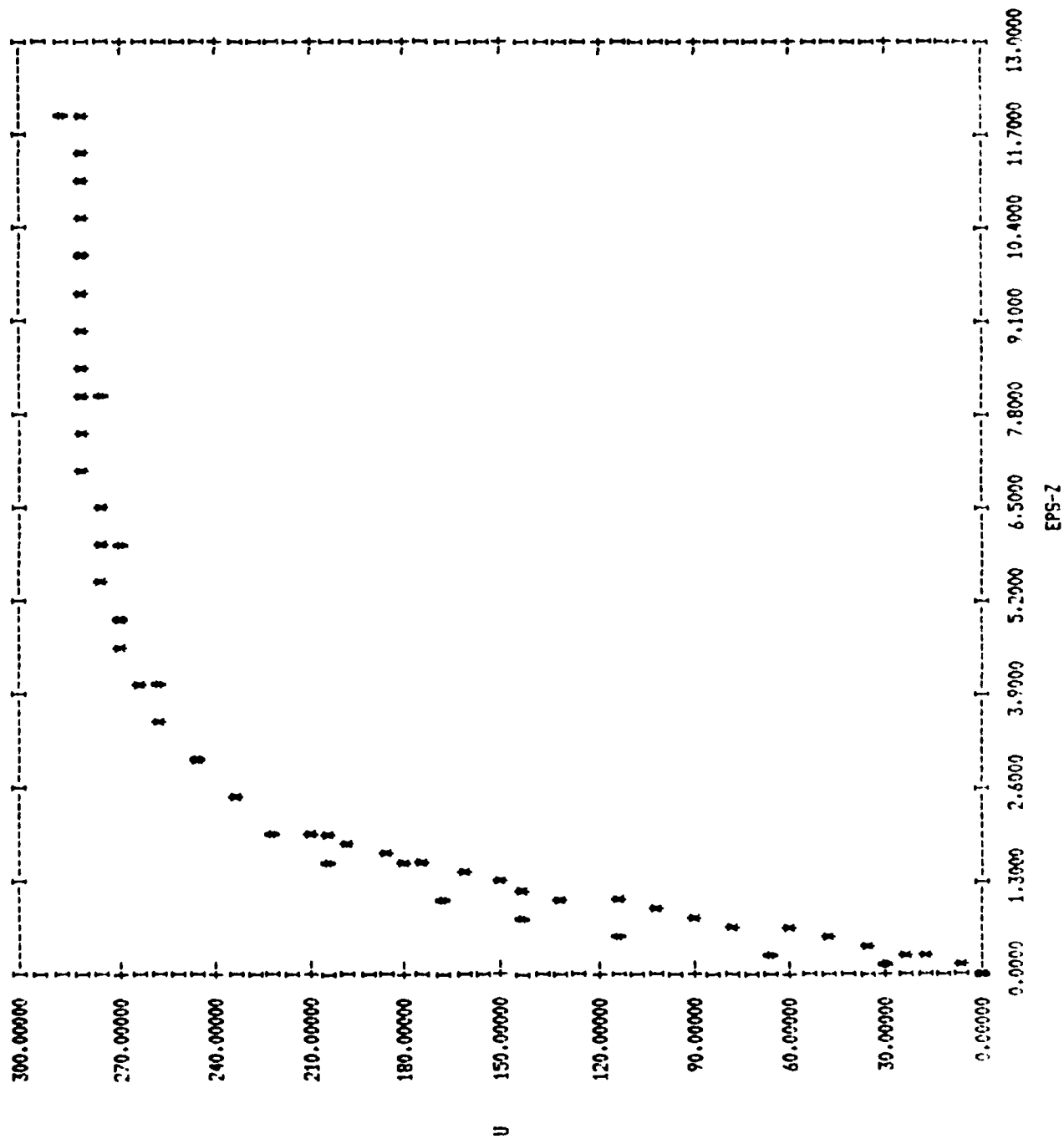| N	EPS-X	EPS-Y	EPS-Z	GAM-XY	GAM-XZ	GAM-YZ	SIG-X	SIG-Y	SIG-Z	TAU-XY	TAU-XZ	TAU-YZ	U	θ-IT
1	-5.0E-04	-5.0E-04	1.0E-03	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	4.10E+02	0.00E-01	0.00E-01	0.00E-01	6.76E+00	2
2	-9.9E-04	-9.9E-04	2.0E-03	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	4.27E+02	0.00E-01	0.00E-01	0.00E-01	1.52E+01	1
3	-1.5E-03	-1.5E-03	3.0E-03	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	4.44E+02	0.00E-01	0.00E-01	0.00E-01	2.53E+01	1
4	-2.0E-03	-2.0E-03	4.0E-03	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	4.60E+02	0.00E-01	0.00E-01	0.00E-01	3.67E+01	1
5	-2.5E-03	-2.5E-03	5.0E-03	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	4.76E+02	0.00E-01	0.00E-01	0.00E-01	4.91E+01	1
6	-3.0E-03	-3.0E-03	6.0E-03	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	4.90E+02	0.00E-01	0.00E-01	0.00E-01	6.24E+01	1
7	-3.5E-03	-3.5E-03	7.0E-03	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.03E+02	0.00E-01	0.00E-01	0.00E-01	7.61E+01	1
8	-4.0E-03	-4.0E-03	8.0E-03	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.15E+02	0.00E-01	0.00E-01	0.00E-01	8.99E+01	1
9	-4.4E-03	-4.4E-03	9.0E-03	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.26E+02	0.00E-01	0.00E-01	0.00E-01	1.03E+02	1
10	-4.9E-03	-4.9E-03	1.0E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.35E+02	0.00E-01	0.00E-01	0.00E-01	1.17E+02	1
11	-5.4E-03	-5.4E-03	1.1E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.43E+02	0.00E-01	0.00E-01	0.00E-01	1.29E+02	1
12	-5.9E-03	-5.9E-03	1.2E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.51E+02	0.00E-01	0.00E-01	0.00E-01	1.41E+02	1
13	-6.4E-03	-6.4E-03	1.3E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.57E+02	0.00E-01	0.00E-01	0.00E-01	1.52E+02	1
14	-6.9E-03	-6.9E-03	1.4E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.62E+02	0.00E-01	0.00E-01	0.00E-01	1.62E+02	1
15	-7.4E-03	-7.4E-03	1.5E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.67E+02	0.00E-01	0.00E-01	0.00E-01	1.72E+02	1
16	-7.9E-03	-7.9E-03	1.6E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.71E+02	0.00E-01	0.00E-01	0.00E-01	1.80E+02	1
17	-8.4E-03	-8.4E-03	1.7E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.74E+02	0.00E-01	0.00E-01	0.00E-01	1.88E+02	1
18	-8.9E-03	-8.9E-03	1.8E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.77E+02	0.00E-01	0.00E-01	0.00E-01	1.95E+02	1
19	-9.4E-03	-9.4E-03	1.9E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.80E+02	0.00E-01	0.00E-01	0.00E-01	2.02E+02	1
20	-9.9E-03	-9.9E-03	2.0E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.82E+02	0.00E-01	0.00E-01	0.00E-01	2.08E+02	1
21	-1.2E-02	-1.2E-02	2.5E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.90E+02	0.00E-01	0.00E-01	0.00E-01	2.31E+02	3
22	-1.5E-02	-1.5E-02	3.0E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.95E+02	0.00E-01	0.00E-01	0.00E-01	2.47E+02	2
23	-1.7E-02	-1.7E-02	3.5E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.97E+02	0.00E-01	0.00E-01	0.00E-01	2.57E+02	2
24	-2.0E-02	-2.0E-02	4.0E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	5.99E+02	0.00E-01	0.00E-01	0.00E-01	2.64E+02	1
25	-2.2E-02	-2.2E-02	4.5E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.00E+02	0.00E-01	0.00E-01	0.00E-01	2.69E+02	1
26	-2.5E-02	-2.5E-02	5.0E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.01E+02	0.00E-01	0.00E-01	0.00E-01	2.73E+02	1
27	-2.7E-02	-2.7E-02	5.5E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.01E+02	0.00E-01	0.00E-01	0.00E-01	2.75E+02	1
28	-3.0E-02	-3.0E-02	6.0E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.02E+02	0.00E-01	0.00E-01	0.00E-01	2.77E+02	1
29	-3.2E-02	-3.2E-02	6.5E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.02E+02	0.00E-01	0.00E-01	0.00E-01	2.79E+02	1
30	-3.5E-02	-3.5E-02	7.0E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.03E+02	0.00E-01	0.00E-01	0.00E-01	2.80E+02	1
31	-3.7E-02	-3.7E-02	7.5E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.03E+02	0.00E-01	0.00E-01	0.00E-01	2.81E+02	1
32	-4.0E-02	-4.0E-02	8.0E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.03E+02	0.00E-01	0.00E-01	0.00E-01	2.82E+02	1
33	-4.2E-02	-4.2E-02	8.5E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.03E+02	0.00E-01	0.00E-01	0.00E-01	2.82E+02	1
34	-4.5E-02	-4.5E-02	9.0E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.03E+02	0.00E-01	0.00E-01	0.00E-01	2.82E+02	1
35	-4.7E-02	-4.7E-02	9.5E-02	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.04E+02	0.00E-01	0.00E-01	0.00E-01	2.83E+02	1
36	-5.0E-02	-5.0E-02	1.0E-01	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.04E+02	0.00E-01	0.00E-01	0.00E-01	2.83E+02	1
37	-5.2E-02	-5.2E-02	1.1E-01	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.04E+02	0.00E-01	0.00E-01	0.00E-01	2.83E+02	1
38	-5.5E-02	-5.5E-02	1.1E-01	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.04E+02	0.00E-01	0.00E-01	0.00E-01	2.83E+02	1
39	-5.7E-02	-5.7E-02	1.2E-01	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.04E+02	0.00E-01	0.00E-01	0.00E-01	2.83E+02	1
40	-6.0E-02	-6.0E-02	1.2E-01	0.0E-01	0.0E-01	0.0E-01	3.92E+02	3.92E+02	6.04E+02	0.00E-01	0.00E-01	0.00E-01	2.83E+02	1

OUTPUT DATA:
MODEL PREDICTIONS -- IN TERMS OF THE KEY RESPONSE
PARAMETERS -- CORRESPONDING TO TEST DATA SET # 1 ARE AS FOLLOWS:

| N	EPS-1	Q	F	U	E-VOL
1	0.10	17.60	391.31	6.76	0.00
2	0.20	34.95	388.61	15.24	0.00
3	0.30	51.94	384.20	25.29	0.00
4	0.40	68.06	378.22	36.67	0.00
5	0.50	93.41	370.87	49.13	0.01
6	0.60	97.73	362.41	62.37	0.01
7	0.70	110.90	353.11	76.06	0.01
8	0.80	122.82	343.29	89.87	0.01
9	0.90	133.48	333.20	103.50	0.01
10	1.00	142.90	323.13	116.70	0.01
11	1.10	151.15	313.29	129.29	0.01
12	1.20	158.32	303.83	141.14	0.01
13	1.30	164.55	294.86	152.18	0.02
14	1.40	169.87	286.45	162.38	0.02
15	1.50	174.48	278.60	171.76	0.02
16	1.60	178.47	271.33	180.35	0.02
17	1.70	181.91	264.62	198.21	0.02
18	1.80	184.90	258.44	195.39	0.02
19	1.90	187.49	252.75	201.95	0.02
20	2.00	189.75	247.52	207.94	0.02
21	2.50	197.75	227.01	231.11	0.02
22	3.00	202.38	212.80	246.86	0.02
23	3.50	205.14	203.10	257.49	0.03
24	4.00	206.73	196.86	264.26	0.03
25	4.50	207.83	192.26	269.22	0.03
26	5.00	208.59	188.92	272.81	0.03
27	5.50	209.27	186.48	275.48	0.03
28	6.00	209.78	184.67	277.46	0.03
29	6.50	210.19	193.32	278.94	0.03
30	7.00	210.49	182.31	280.05	0.03
31	7.50	210.74	181.56	280.89	0.03
32	8.00	210.92	180.99	281.52	0.03
33	8.50	211.06	180.56	281.99	0.03
34	9.00	211.16	180.24	292.35	0.03
35	9.50	211.25	179.99	292.62	0.03
36	10.00	211.31	179.81	282.83	0.03
37	10.50	211.35	179.67	282.98	0.03
38	11.00	211.39	179.56	283.10	0.03
39	11.50	211.42	179.48	283.19	0.03
40	12.00	211.44	179.42	283.25	0.03
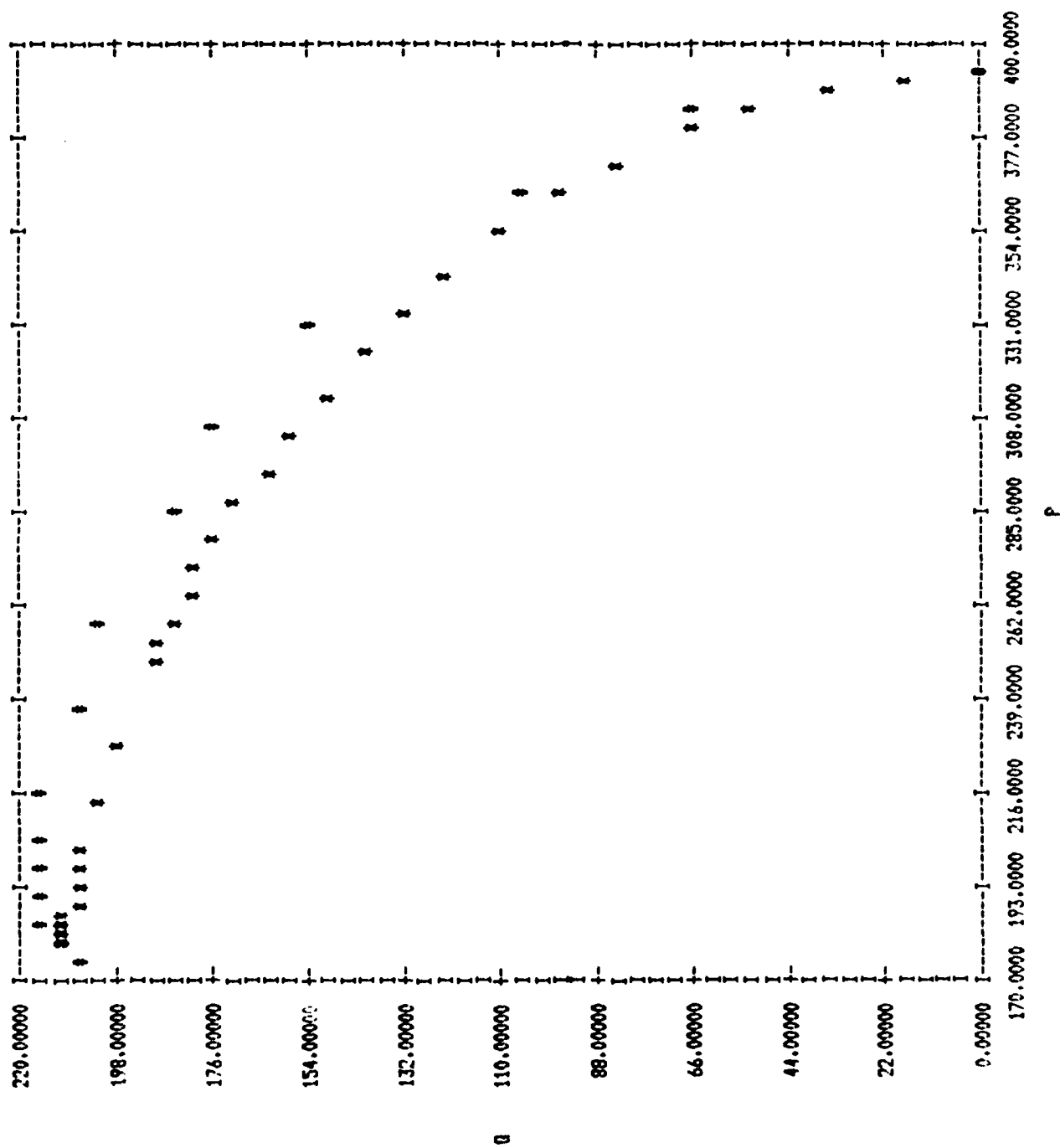
EPS-Z

84

EPS-Z

U

85

OUTPUT DATA:
MODEL PREDICTIONS CORRESPONDING TO TEST DATA SET # 2 ARE AS FOLLOWS:

N	EPS-X	EPS-Y	EPS-Z	GAM-XY	GAM-XZ	GAM-YZ	SIG-X	SIG-Y	SIG-Z	TAU-XY	TAU-XZ	TAU-YZ	U	#-IT
1	-5.0E-04	-5.0E-04	1.0E-03	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	2.13E+02	0.00E-01	0.00E-01	0.00E-01	5.98E+00	3
2	-9.9E-04	-9.9E-04	2.0E-03	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	2.29E+02	0.00E-01	0.00E-01	0.00E-01	1.29E+01	2
3	-1.5E-03	-1.5E-03	3.0E-03	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	2.44E+02	0.00E-01	0.00E-01	0.00E-01	1.78E+01	2
4	-2.0E-03	-2.0E-03	4.0E-03	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	2.58E+02	0.00E-01	0.00E-01	0.00E-01	2.33E+01	2
5	-2.5E-03	-2.5E-03	5.0E-03	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	2.70E+02	0.00E-01	0.00E-01	0.00E-01	2.85E+01	2
6	-3.0E-03	-3.0E-03	6.0E-03	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	2.82E+02	0.00E-01	0.00E-01	0.00E-01	3.33E+01	2
7	-3.5E-03	-3.5E-03	8.0E-03	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	2.92E+02	0.00E-01	0.00E-01	0.00E-01	3.78E+01	2
8	-4.0E-03	-4.0E-03	8.0E-03	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.02E+02	0.00E-01	0.00E-01	0.00E-01	4.21E+01	2
9	-4.5E-03	-4.5E-03	9.0E-03	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.10E+02	0.00E-01	0.00E-01	0.00E-01	4.60E+01	2
10	-5.0E-03	-5.0E-03	1.0E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.18E+02	0.00E-01	0.00E-01	0.00E-01	4.97E+01	2
11	-5.5E-03	-5.5E-03	1.1E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.25E+02	0.00E-01	0.00E-01	0.00E-01	5.31E+01	1
12	-6.0E-03	-6.0E-03	1.2E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.31E+02	0.00E-01	0.00E-01	0.00E-01	5.62E+01	1
13	-6.5E-03	-6.5E-03	1.3E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.37E+02	0.00E-01	0.00E-01	0.00E-01	5.92E+01	1
14	-7.0E-03	-7.0E-03	1.4E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.42E+02	0.00E-01	0.00E-01	0.00E-01	6.19E+01	1
15	-7.5E-03	-7.5E-03	1.5E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.47E+02	0.00E-01	0.00E-01	0.00E-01	6.44E+01	1
16	-8.0E-03	-8.0E-03	1.6E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.51E+02	0.00E-01	0.00E-01	0.00E-01	6.67E+01	1
17	-8.5E-03	-8.5E-03	1.7E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.55E+02	0.00E-01	0.00E-01	0.00E-01	6.89E+01	1
18	-9.0E-03	-9.0E-03	1.8E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.58E+02	0.00E-01	0.00E-01	0.00E-01	7.09E+01	1
19	-9.5E-03	-9.5E-03	1.9E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.61E+02	0.00E-01	0.00E-01	0.00E-01	7.27E+01	1
20	-1.0E-02	-1.0E-02	2.0E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.63E+02	0.00E-01	0.00E-01	0.00E-01	7.45E+01	1
21	-1.2E-02	-1.2E-02	2.5E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.73E+02	0.00E-01	0.00E-01	0.00E-01	8.14E+01	5
22	-1.5E-02	-1.5E-02	3.0E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.79E+02	0.00E-01	0.00E-01	0.00E-01	8.61E+01	3
23	-1.7E-02	-1.7E-02	3.5E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.82E+02	0.00E-01	0.00E-01	0.00E-01	8.94E+01	3
24	-2.0E-02	-2.0E-02	4.0E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.84E+02	0.00E-01	0.00E-01	0.00E-01	9.18E+01	2
25	-2.2E-02	-2.2E-02	4.5E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.85E+02	0.00E-01	0.00E-01	0.00E-01	9.35E+01	1
26	-2.5E-02	-2.5E-02	5.0E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.86E+02	0.00E-01	0.00E-01	0.00E-01	9.46E+01	1
27	-2.7E-02	-2.7E-02	5.5E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.86E+02	0.00E-01	0.00E-01	0.00E-01	9.55E+01	1
28	-3.0E-02	-3.0E-02	6.0E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.86E+02	0.00E-01	0.00E-01	0.00E-01	9.61E+01	1
29	-3.2E-02	-3.2E-02	6.5E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.86E+02	0.00E-01	0.00E-01	0.00E-01	9.66E+01	1
30	-3.5E-02	-3.5E-02	7.0E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.69E+01	1
31	-3.7E-02	-3.7E-02	7.5E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.72E+01	1
32	-4.0E-02	-4.0E-02	8.0E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.74E+01	1
33	-4.2E-02	-4.2E-02	8.5E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.75E+01	1
34	-4.5E-02	-4.5E-02	9.0E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.97E+02	0.00E-01	0.00E-01	0.00E-01	9.76E+01	1
35	-4.7E-02	-4.7E-02	9.5E-02	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.77E+01	1
36	-5.0E-02	-5.0E-02	1.0E-01	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.78E+01	1
37	-5.2E-02	-5.2E-02	1.1E-01	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.78E+01	1
38	-5.5E-02	-5.5E-02	1.1E-01	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.79E+01	1
39	-5.7E-02	-5.7E-02	1.2E-01	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.79E+01	1
40	-6.0E-02	-6.0E-02	1.3E-01	0.0E-01	0.0E-01	0.0E-01	1.96E+02	1.96E+02	3.87E+02	0.00E-01	0.00E-01	0.00E-01	9.79E+01	1

86

OUTPUT DATA:
MODEL PREDICTIONS -- IN TERMS OF THE KEY RESPONSE
PARAMETERS -- CORRESPONDING TO TEST DATA SET # 2 ARE AS FOLLOWS:

N	EPS-1	Q	P	U	E-VOL
1	0.10	17.21	195.86	5.98	0.00
2	0.20	33.31	195.16	12.04	0.00
3	0.30	48.15	194.31	17.84	0.00
4	0.40	61.78	193.37	23.32	0.00
5	0.50	74.26	192.38	28.47	0.00
6	0.60	85.67	191.35	33.31	0.00
7	0.70	96.07	190.29	37.83	0.00
8	0.80	105.52	189.22	42.05	0.00
9	0.90	114.10	188.14	45.99	0.00
10	1.00	121.87	187.06	49.66	0.00
11	1.10	128.89	186.00	53.07	0.01
12	1.20	135.22	184.94	56.23	0.01
13	1.30	140.89	183.91	59.18	0.01
14	1.40	146.00	182.89	61.87	0.01
15	1.50	150.59	181.91	64.39	0.01
16	1.60	154.72	180.95	66.72	0.01
17	1.70	158.43	180.03	68.88	0.01
18	1.80	161.75	179.13	70.88	0.01
19	1.90	164.73	178.27	72.74	0.01
20	2.00	167.40	177.45	74.45	0.01
21	2.50	177.40	173.81	81.42	0.01
22	3.00	182.85	170.90	86.15	0.01
23	3.50	185.98	168.65	89.44	0.01
24	4.00	187.93	166.93	91.81	0.01
25	4.50	189.03	165.62	93.49	0.01
26	5.00	189.45	164.66	94.60	0.01
27	5.50	189.90	163.94	95.46	0.01
28	6.00	190.13	163.39	96.08	0.01
29	6.50	190.33	162.99	96.56	0.01
30	7.00	190.46	162.68	96.91	0.01
31	7.50	190.56	162.44	97.18	0.01
32	8.00	190.63	162.27	97.38	0.01
33	8.50	190.68	162.13	97.53	0.01
34	9.00	190.72	162.03	97.64	0.01
35	9.50	190.75	161.96	97.73	0.01
36	10.00	190.77	161.90	97.79	0.01
37	10.50	190.78	161.86	97.84	0.01
38	11.00	190.80	161.82	97.88	0.01
39	11.50	190.80	161.80	97.90	0.01
40	12.00	190.81	161.78	97.92	0.01
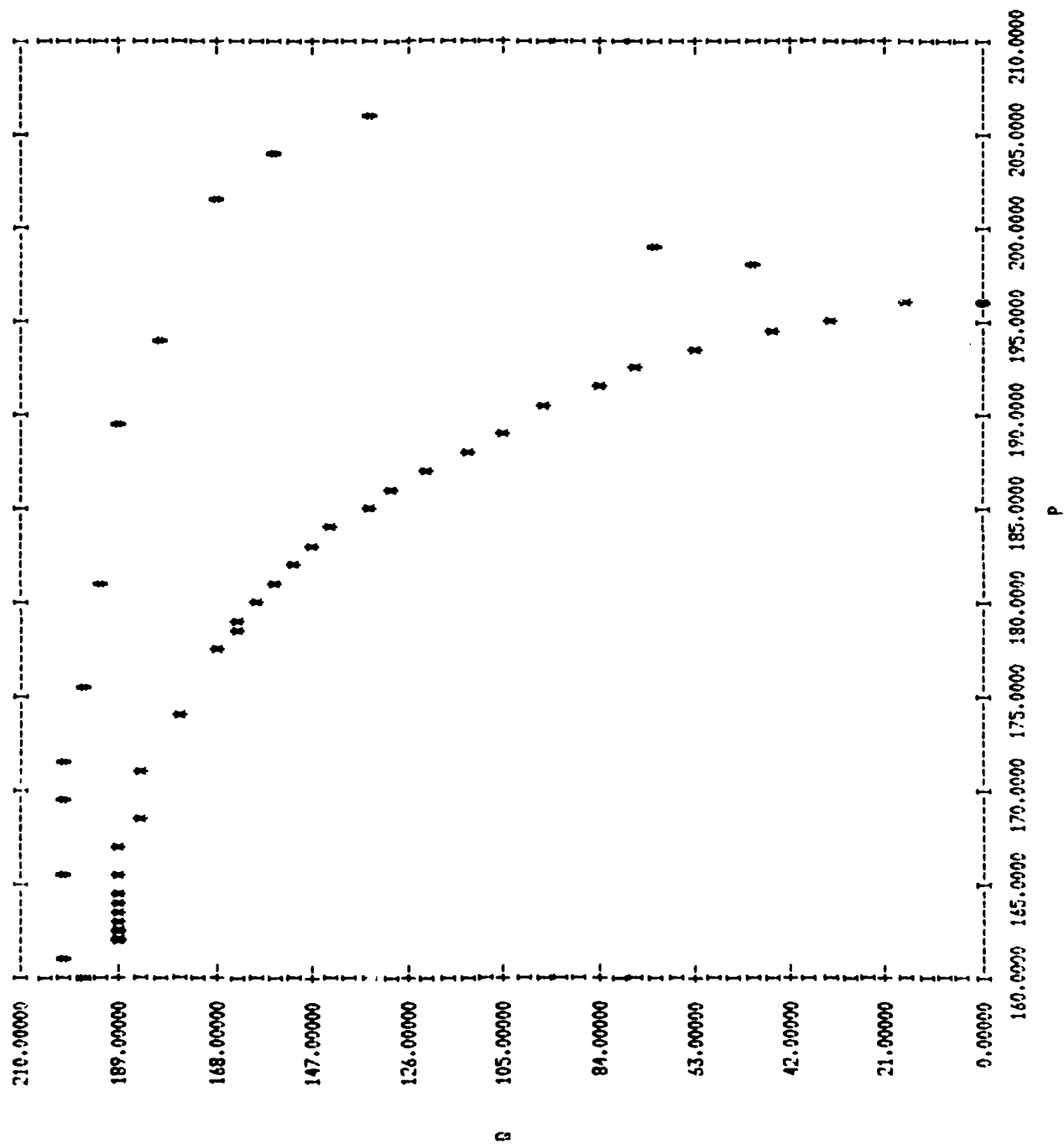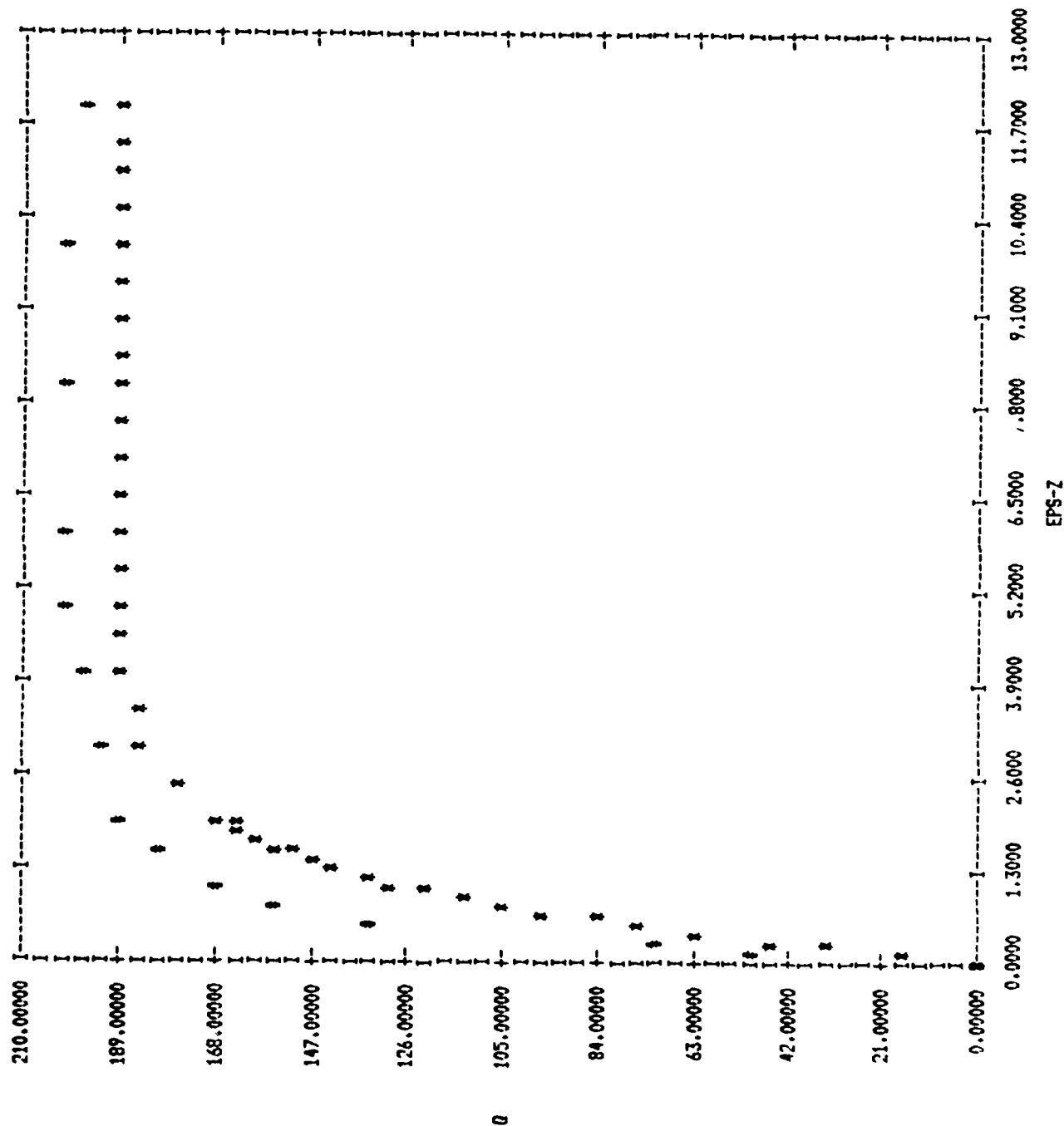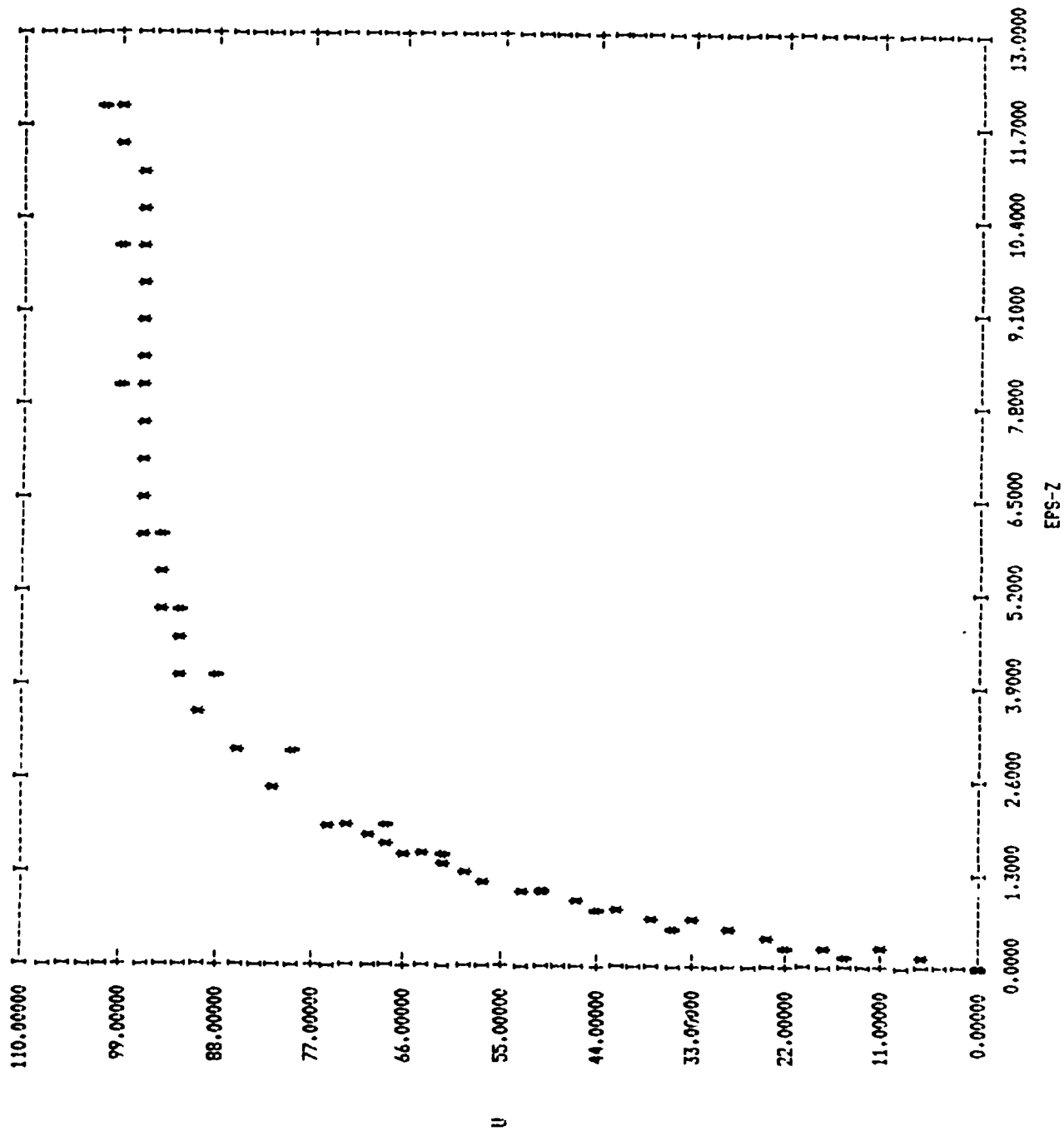
EPS-Z

89

EPS-Z

OUTPUT DATA:
MODEL PREDICTIONS CORRESPONDING TO TEST DATA SET # 3 ARE AS FOLLOWS:

N	EPS-X	EPS-Y	EPS-Z	GAM-XY	GAM-XZ	GAM-YZ	SIG-X	SIG-Y	SIG-Z	TAU-XY	TAU-XZ	TAU-YZ	U	#-IT
1	-5.0E-04	-5.0E-04	1.0E-03	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	8.24E+01	0.00E-01	0.00E-01	0.00E-01	4.55E+00	3
2	-1.0E-03	-1.0E-03	2.0E-03	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	9.62E+01	0.00E-01	0.00E-01	0.00E-01	6.07E+00	3
3	-1.5E-03	-1.5E-03	3.0E-03	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.08E+02	0.00E-01	0.00E-01	0.00E-01	5.99E+00	2
4	-2.0E-03	-2.0E-03	4.0E-03	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.19E+02	0.00E-01	0.00E-01	0.00E-01	5.97E+00	1
5	-2.5E-03	-2.5E-03	5.0E-03	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.30E+02	0.00E-01	0.00E-01	0.00E-01	5.76E+00	2
6	-3.0E-03	-3.0E-03	6.0E-03	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.41E+02	0.00E-01	0.00E-01	0.00E-01	5.29E+00	2
7	-3.5E-03	-3.5E-03	7.0E-03	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.51E+02	0.00E-01	0.00E-01	0.00E-01	4.51E+00	2
8	-4.0E-03	-4.0E-03	8.0E-03	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.61E+02	0.00E-01	0.00E-01	0.00E-01	3.43E+00	2
9	-4.5E-03	-4.5E-03	9.0E-03	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.70E+02	0.00E-01	0.00E-01	0.00E-01	2.03E+00	2
10	-5.0E-03	-5.0E-03	1.0E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.79E+02	0.00E-01	0.00E-01	0.00E-01	3.00E-01	2
11	-5.5E-03	-5.5E-03	1.1E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.87E+02	0.00E-01	0.00E-01	0.00E-01	-1.82E+00	2
12	-6.0E-03	-6.0E-03	1.2E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	1.94E+02	0.00E-01	0.00E-01	0.00E-01	-4.34E+00	2
13	-6.5E-03	-6.5E-03	1.3E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.01E+02	0.00E-01	0.00E-01	0.00E-01	-7.20E+00	2
14	-7.0E-03	-7.0E-03	1.4E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.06E+02	0.00E-01	0.00E-01	0.00E-01	-1.03E+01	2
15	-7.5E-03	-7.5E-03	1.5E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.10E+02	0.00E-01	0.00E-01	0.00E-01	-1.33E+01	2
16	-8.0E-03	-8.0E-03	1.6E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.13E+02	0.00E-01	0.00E-01	0.00E-01	-1.61E+01	2
17	-8.5E-03	-8.5E-03	1.7E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.16E+02	0.00E-01	0.00E-01	0.00E-01	-1.84E+01	2
18	-9.0E-03	-9.0E-03	1.9E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.18E+02	0.00E-01	0.00E-01	0.00E-01	-2.01E+01	2
19	-9.5E-03	-9.5E-03	1.9E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.19E+02	0.00E-01	0.00E-01	0.00E-01	-2.12E+01	2
20	-1.0E-02	-1.0E-02	2.0E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.51E+01	2.20E+02	0.00E-01	0.00E-01	0.00E-01	-2.17E+01	1
21	-1.3E-02	-1.3E-02	2.5E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.25E+02	0.00E-01	0.00E-01	0.00E-01	-2.23E+01	5
22	-1.5E-02	-1.5E-02	3.0E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.28E+02	0.00E-01	0.00E-01	0.00E-01	-2.14E+01	3
23	-1.8E-02	-1.8E-02	3.5E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.29E+02	0.00E-01	0.00E-01	0.00E-01	-2.08E+01	2
24	-2.0E-02	-2.0E-02	4.0E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.30E+02	0.00E-01	0.00E-01	0.00E-01	-2.04E+01	2
25	-2.3E-02	-2.3E-02	4.5E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.51E+01	2.30E+02	0.00E-01	0.00E-01	0.00E-01	-2.01E+01	2
26	-2.5E-02	-2.5E-02	5.0E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.30E+02	0.00E-01	0.00E-01	0.00E-01	-2.01E+01	1
27	-2.8E-02	-2.8E-02	5.5E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.99E+01	1
28	-3.0E-02	-3.0E-02	6.0E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.98E+01	1
29	-3.3E-02	-3.3E-02	6.5E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.97E+01	1
30	-3.5E-02	-3.5E-02	7.0E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.97E+01	1
31	-3.8E-02	-3.8E-02	7.5E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.51E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.97E+01	1
32	-4.0E-02	-4.0E-02	8.0E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.97E+01	1
33	-4.3E-02	-4.3E-02	8.5E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.96E+01	1
34	-4.5E-02	-4.5E-02	9.0E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.96E+01	1
35	-4.8E-02	-4.8E-02	9.5E-02	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.96E+01	1
36	-5.0E-02	-5.0E-02	1.0E-01	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.96E+01	1
37	-5.3E-02	-5.3E-02	1.1E-01	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.96E+01	1
38	-5.8E-02	-5.8E-02	1.2E-01	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.96E+01	1
39	-5.8E-02	-5.8E-02	1.2E-01	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.96E+01	1
40	-6.0E-02	-6.0E-02	1.2E-01	0.0E-01	0.0E-01	0.0E-01	6.54E+01	6.54E+01	2.31E+02	0.00E-01	0.00E-01	0.00E-01	-1.96E+01	1

91

END

FILMED

DTIC

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

OUTPUT DATA:

MODEL PREDICTIONS -- IN TERMS OF THE KEY RESPONSE
PARAMETERS -- CORRESPONDING TO TEST DATA SET # 3 ARE AS FOLLOWS:

N	EPS-1	Q	F	U	E-VOL
1	0.10	17.00	66.49	4.55	0.00
2	0.20	30.87	69.60	6.07	0.00
3	0.30	42.33	73.49	5.99	0.00
4	0.40	53.69	77.30	5.97	0.00
5	0.50	64.77	81.21	5.76	0.00
6	0.60	75.49	85.26	5.29	0.00
7	0.70	85.77	89.46	4.51	0.00
8	0.80	95.56	93.81	3.43	0.00
9	0.90	104.81	98.29	2.03	0.00
10	1.00	113.46	102.90	0.30	0.00
11	1.10	121.46	107.69	-1.82	0.00
12	1.20	128.74	112.64	-4.34	0.00
13	1.30	135.14	117.64	-7.20	0.00
14	1.40	140.52	122.50	-10.27	0.00
15	1.50	144.79	126.99	-13.34	0.00
16	1.60	148.02	130.86	-16.12	0.00
17	1.70	150.42	133.93	-18.40	0.00
18	1.80	152.26	136.22	-20.07	0.00
19	1.90	153.74	137.84	-21.19	0.00
20	2.00	155.00	138.81	-21.74	0.00
21	2.50	159.64	140.93	-22.32	0.00
22	3.00	162.15	140.82	-21.37	0.00
23	3.50	163.54	140.70	-20.78	0.00
24	4.00	164.38	140.58	-20.39	0.00
25	4.50	164.82	140.47	-20.13	0.00
26	5.00	164.96	140.38	-20.00	0.00
27	5.50	165.10	140.31	-19.88	0.00
28	6.00	165.16	140.26	-19.81	0.00
29	6.50	165.22	140.22	-19.75	0.00
30	7.00	165.24	140.19	-19.71	0.00
31	7.50	165.27	140.16	-19.68	0.00
32	8.00	165.28	140.15	-19.65	0.00
33	8.50	165.29	140.13	-19.64	0.00
34	9.00	165.29	140.12	-19.62	0.00
35	9.50	165.30	140.11	-19.62	0.00
36	10.00	165.30	140.11	-19.61	0.00
37	10.50	165.30	140.10	-19.60	0.00
38	11.00	165.30	140.10	-19.60	0.00
39	11.50	165.30	140.10	-19.60	0.00
40	12.00	165.31	140.10	-19.59	0.00

EPS-Z

95
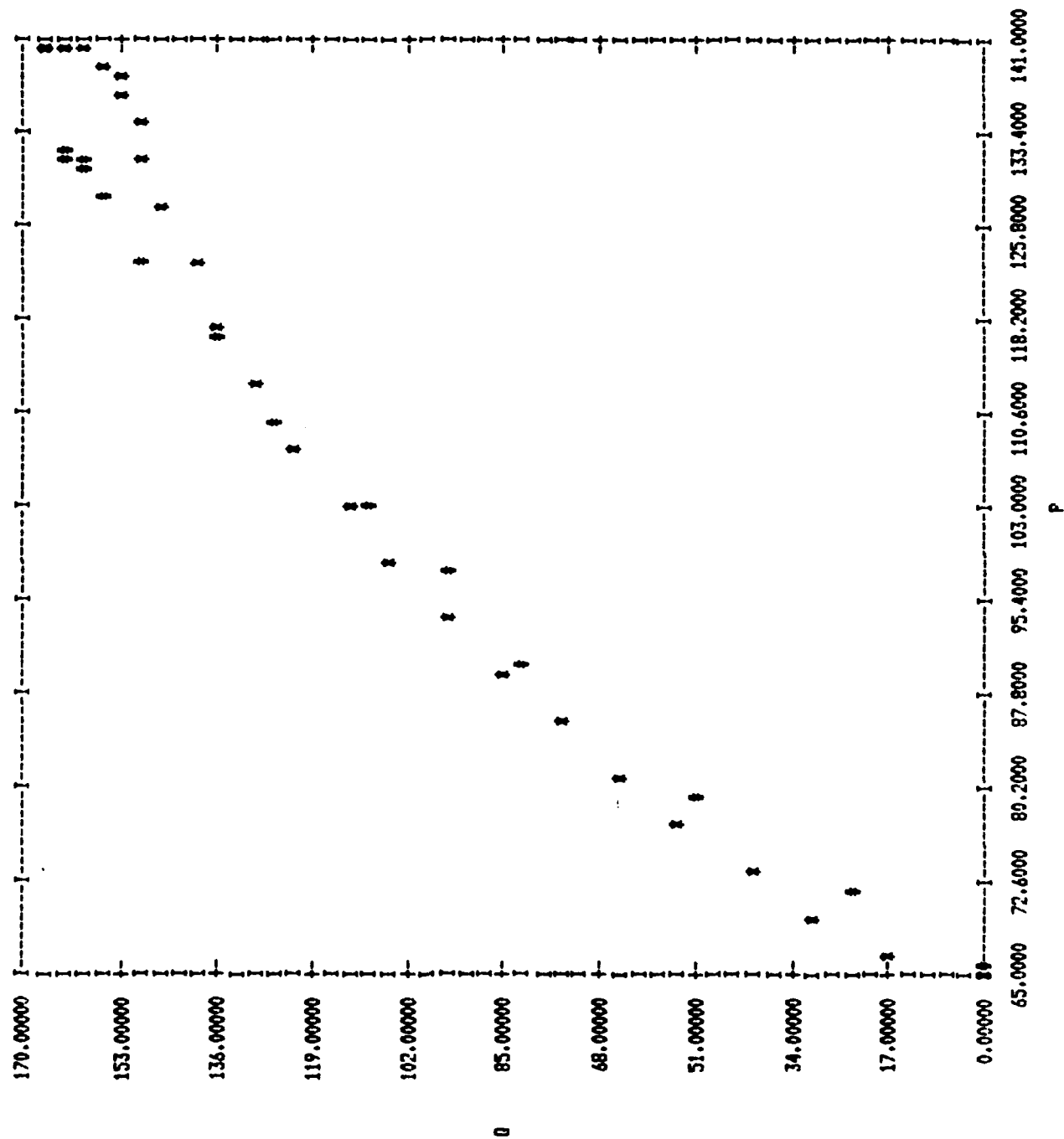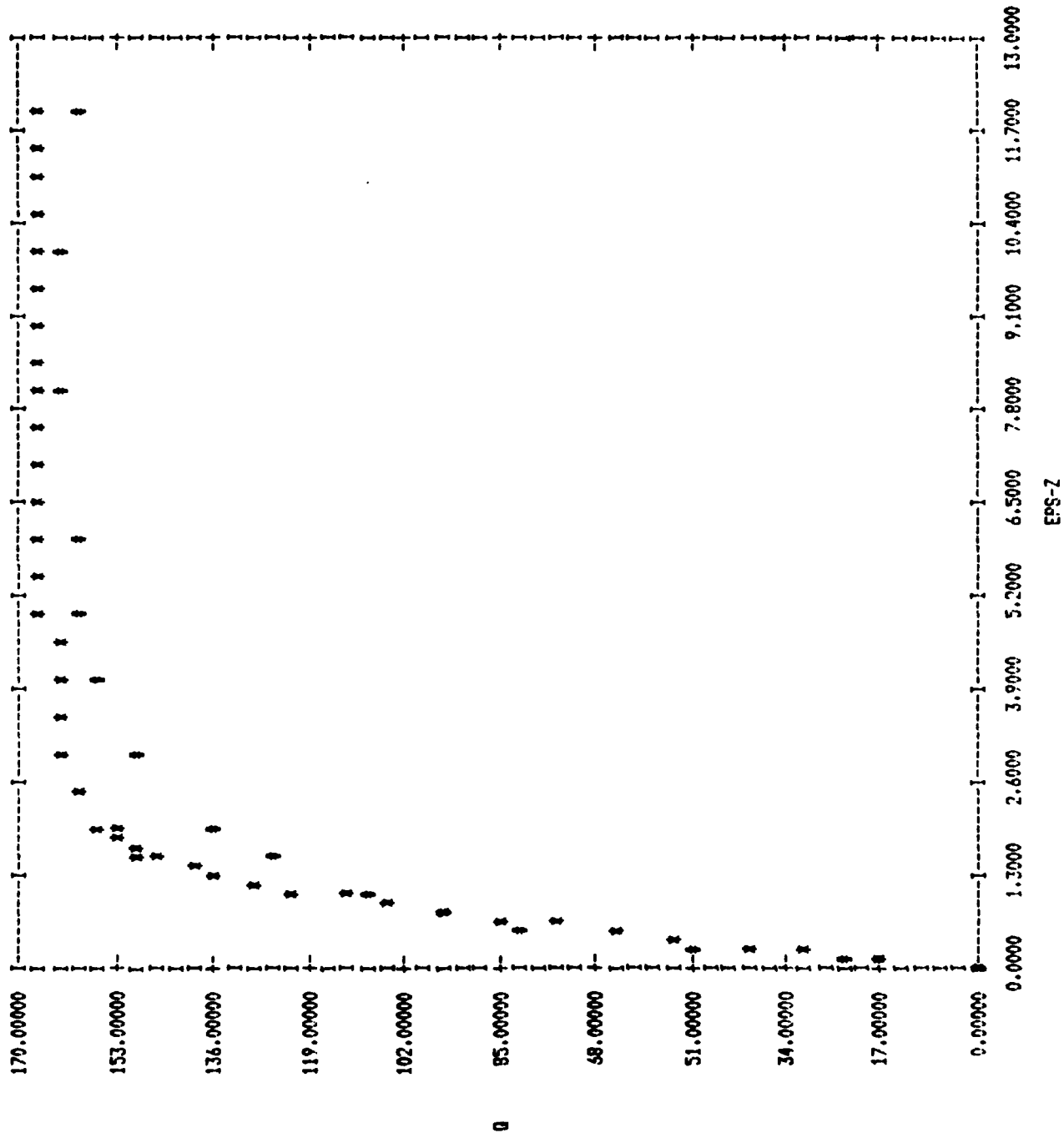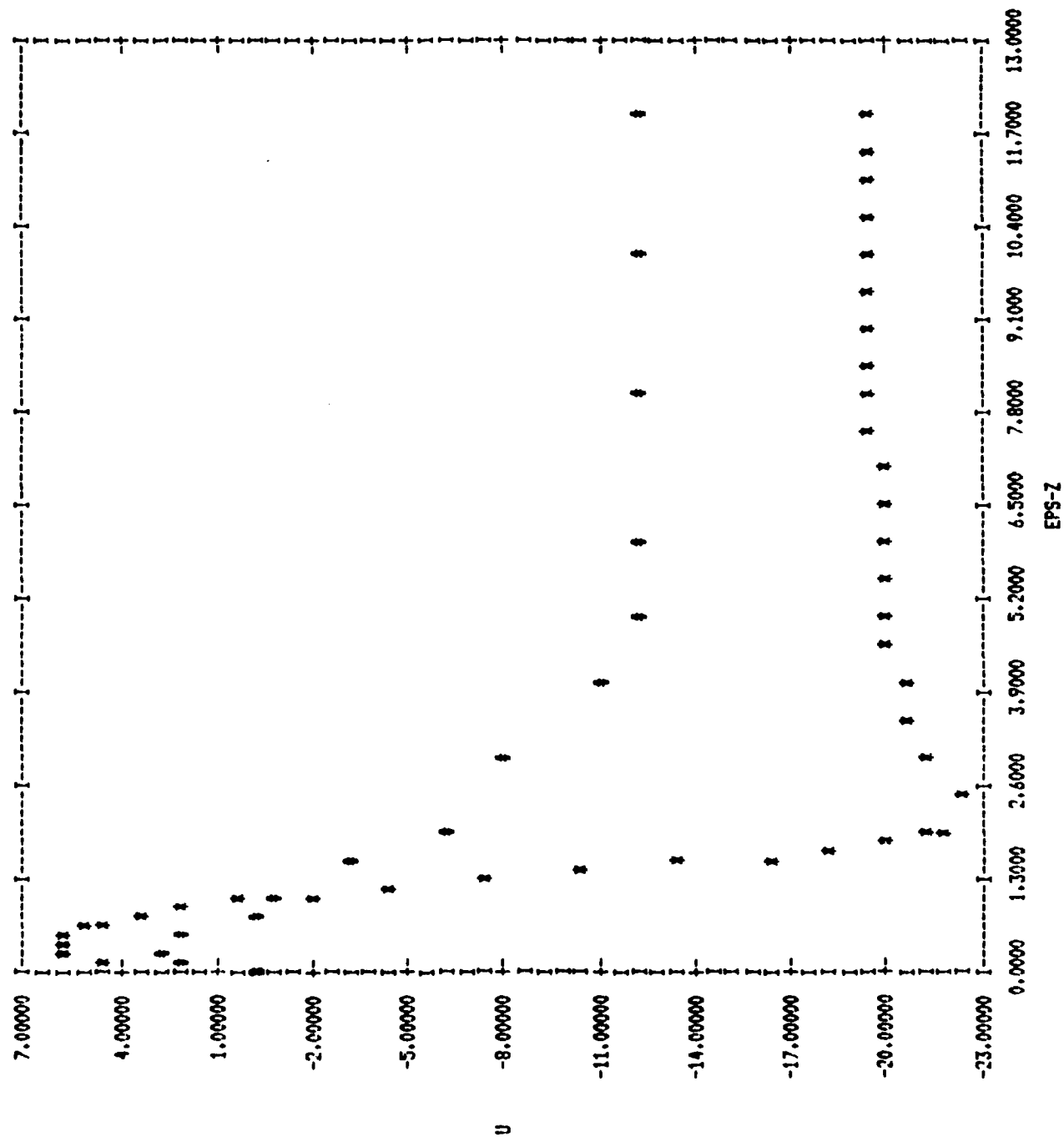
IX. PROGRAM LISTING

MODCAL

Bounding Surface Soil Plasticity Model Calibration

and Prediction Code

by

J. S. DeNatale

Department of Civil Engineering

University of California, Davis

November 1982

```
C      ************************
C      *                      *
C      *        MODCAL        *
C      *                      *
C      ************************
C
C      THIS SUBROUTINE SERVES AS THE MAIN DRIVING
C      PROGRAM FOR THE BOUNDING SURFACE SOIL PLASTICITY
C      MODEL CALIBRATION AND PREDICTION CODE.
C
C      WRITTEN BY J.S. DE NATALE,
C      DEPARTMENT OF CIVIL ENGINEERING,
C      UNIVERSITY OF CALIFORNIA, DAVIS.
C      VERSION I: MAY 1982.
C
       DIMENSION XV(17)
       CALL OPEN
       CALL DATAIN(JOPT,JRUN)
       IF(JOPT.NE.0) CALL BSMOPT(XV)
       IF(JRUN.NE.0) CALL EVAL(XV,FV,1)
       CALL EXIT
       END
C
C
C
       SUBROUTINE DATAIN(IOPT,IRUN)
C
C      THIS SUBROUTINE READS IN ALL VALUES REQUIRED TO CALIBRATE
C      AND/OR USE THE BOUNDING SURFACE PLASTICITY MODEL PROGRAMS.
C
C      WRITTEN BY J.S. DE NATALE,
C      DEPARTMENT OF CIVIL ENGINEERING,
C      UNIVERSITY OF CALIFORNIA, DAVIS.
C      VERSION I: MAY 1982.
C
       COMMON/BLK7/SFUN,SF(17),XL(17),XU(17)
       COMMON/BLKA/NTST,JOPT,JPLT,NOPT,KOPT(17)
       COMMON/BLKB/W1(6),W2(6,3),W3(6,3,20)
       COMMON/BLKC/PRP1(19),PRP2(6,4),IDAT(3),RDAT(2),
      *            NSEG(6),LTYP(6,4,7),VALU(6,4,7)
       COMMON/BLKD/NPLT(6),KPLT(6,3),NEXP(6,3),
      *            XV(6,3,20),YV(6,3,20),PINC(61,5)
       COMMON/BLKE/KRSL,POWR,XMXR(5),YMXR(5),XMXP(6,3),
      *            YMXP(6,3),PLIM,TLIM
       DIMENSION   TITL(20)
C
C      FORMAT STATEMENTS
C
  800 FORMAT(20A4)
  804 FORMAT(16I5)
  808 FORMAT(8E10.3)
  812 FORMAT(1I5,5X,4E10.3)
  816 FORMAT(3I5,5X,4E10.3)
  820 FORMAT(4(2I5,E10.3))
```

97

```
824 FORMAT(6(I1,E9.2),5X,I5,E10.3)
828 FORMAT(4(2I3,I4,E10.3))
900 FORMAT(1H1/5X,20A4)
904 FORMAT(//5X,'INPUT DATA:'/5X,'CONTROL CODES:'/
    *          5X,'---------------'//
    *          15X,'JOPT =',I3/
    *          15X,'JRUN =',I3/
    *          15X,'JPLT =',I3/
    *          15X,'NTST =',I3/
    *          15X,'KRSL =',I3/
    *          15X,'KNRM =',I3/
    *          15X,'POWR =',1PE10.2/
    *          15X,'PLIM =',  E10.2/
    *          15X,'TLIM =',  E10.2/
    *          15X,'NOPT =',I3/
    *          15X,'KOPT =',15(I3,3X))
908 FORMAT(//5X,'INPUT DATA:'/5X,'MATERIAL PROPERTIES:'/
    *          5X,'--------------------'//
    *          15X,'LAMBDA                =',1PE10.3/
    *          15X,'KAPPA                 =',  E10.3/
    *          15X,'MUC (CSL SLOPE)       =',  E10.3/
    *          15X,'MUE/MUC               =',  E10.3/
    *          15X,'SHEAR MODULUS  G      =',  E10.3/
    *          15X,'CBULK MODULUS  GAMMA  =',  E10.3/
    *          15X,'TRANSITIONAL STRESS PL =', E10.3/
    *          15X,'ATMOSPHERIC PRESSURE PA =', E10.3///
    *          5X,'MODEL CONSTANTS:'/5X,'-----------------'//
    *          15X,'RC    =',E10.3/15X,'AC     =',E10.3/
    *          15X,'T     =',E10.3/15X,'RE/RC  =',E10.3/
    *          15X,'AE/AC =',E10.3/
    *          15X,'PROJECTION POINT C =',E10.3/
    *          15X,'ELASTIC  NUCLEUS S =',E10.3/
    *          15X,'HC    =',E10.3/15X,'MC     =',E10.3/
    *          15X,'HE/HC =',E10.3/15X,'ME/MC  =',E10.3)
912 FORMAT(//5X,'INITIAL STATE PROPERTIES:'/
    *          5X,'--------------------------'//
    *          15X,'TEST',5X,'OCR',2X,'VOID-R',4X,'P-PRECON',
    *          4X,'P-CONFIN'/15X,'----',5X,'---',2X,'------',
    *          2(4X,'--------')/)
916 FORMAT(15X,I4,2F8.2,1P3E12.3)
920 FORMAT(//5X,'INPUT DATA:'/5X,'ITERATION INFORMATION ',
    *          'AND ANALYTICAL OPTIONS:'/
    *          5X,'-----------------------------------------------'//
    *          15X,'MAX # OF ITERATIONS  =',I3/
    *          15X,'MAX RELATIVE ERRORS  =',1PE10.3/
    *          15X,'ACCELERATION FACTOR LIMITS  =',E10.3)
924 FORMAT(/10X,'**** DRAINED CONDITIONS ****')
928 FORMAT(/10X,'**** UNDRAINED CONDITIONS ****')
932 FORMAT(/10X,'**** ENGINEERING STRESSES AND STRAINS ASSUMED ****')
936 FORMAT(/10X,'**** TRUE STRESSES AND NATURAL STRAINS ASSUMED ****')
940 FORMAT(/10X,'**** REFORMULATED NEARLY INCOMPRESSIBLE ',
    *          'ANALYSIS ****')
944 FORMAT(/10X,'**** NON-REFORMULATED NEARLY INCOMPRESSIBLE ',
    *          'ANALYSIS ****')
```

```
  948 FORMAT(//5X,'INPUT DATA:'/5X,'SPECIFIED LOADING HISTORIES:'/
     *          5X,'------------------------------'//
     *          5X,'TEST',2X,'SEG-#',2X,'#-INC',2X,'S-RATIO',
     *          2X,'SS-CODE',3X,'SIG/EPS-X',3X,'SIG/EPS-Y',
     *          3X,'SIG/EPS-Z',2X,'TAU/GAM-XY',2X,'TAU/GAM-XY',
     *          2X,'TAU/GAM-YZ'/5X,'----',2(2X,'-----'),
     *          2(2X,'-------'),3(3X,'---------'),3(2X,'----------')/)
  952 FORMAT(5X,I4,2(2X,I5),F9.2,3X,6I1,1P6E12.3)
  956 FORMAT(20X)
C
C     READ HEADING INFORMATION AND CONTROL CODES
C
      READ(5,800) TITL
      READ(5,804) NTST,JOPT,JRUN,JPLT,KRSL,KNRM,NOPT,
     *          (KOPT(II),II=1,NOPT)
      IOPT=JOPT
      IRUN=JRUN
      IF(KNRM.EQ.0) POWR=1.00
      IF(KNRM.NE.0) POWR=2.00
C
C     READ MATERIAL AND MODEL PROPERTIES
C
      READ(5,808) (PRP1(II),II= 1,15)
      READ(5,808) (PRP1(II),II=16,19)
      IF(PRP1(14).GT.1.00) PRP1(14)=1.00
      IF(PRP1(15).LT.1.00) PRP1(15)=1.00
      DO 110 II=1,NTST
  110 READ(5,812) ITST,(PRP2(ITST,JJ),JJ=1,4)
C
C     READ CONVERGENCE CRITERIA AND ITERATION INFORMATION
C
      READ(5,816) (IDAT(II),II=1,3),(RDAT(II),II=1,2),PLIM,TLIM
C
C     READ SPECIFIED LOADING HISTORIES
C
      DO 120 II=1,NTST
      READ(5,804) ITST,NSEG(II)
      DO 120 JJ=1,NSFG(II)
      READ(5,824) (LTYP(II,JJ,KK),VALU(II,JJ,KK),KK=1,7)
  120 IF(VALU(II,JJ,7).EQ.0.0) VALU(II,JJ,7)=1.0
C
C     READ EXPERIMENTAL DATA AND PLOTTING INSTRUCTIONS
C
      NSWT=0
      DO 140 II=1,NTST
      READ(5,820) ITST,NPLT(II),W1(II),NWPT
      READ(5,820) ((KPLT(II,KK),NEXP(II,KK),W2(II,KK)),
     *          KK=1,NPLT(II))
      NSWT=NSWT+NWPT
      DO 130 JJ=1,NPLT(II)
      IF(NEXP(II,JJ).EQ.0) GO TO 140
      READ(5,808) (YV(II,JJ,KK),KK=1,NEXP(II,JJ))
      READ(5,808) (XV(II,JJ,KK),KK=1,NEXP(II,JJ))
  130 CONTINUE
```

```fortran
  140 CONTINUE
      DO 150 II=1,NTST
      IF(W1(II).EQ.0.0) W1(II)=1.0
      DO 150 JJ=1,NPLT(II)
      IF(W2(II,JJ).EQ.0.0) W2(II,JJ)=1.0
      DO 150 KK=1,NEXP(II,JJ)
  150 W3(II,JJ,KK)=1.0
C
C     READ SPECIAL EXPERIMENTAL WEIGHTINGS
C
      IF(NSWT.EQ.0) GO TO 170
      A1=FLOAT(NSWT)/4.0
      A2=AINT(A1)
      IF(A1-A2.GT.0.0) A2=A2+1.0
      NW=IFIX(A2)
      NX=NSWT-4*(NW-1)
      DO 160 II=1,NW
      IF(II.LT.NW) READ(5,828) ((IT,IP,IE,
     *                          W3(IT,IP,IE)),JJ=1, 4)
  160 IF(II.EQ.NW) READ(5,828) ((IT,IP,IE,
     *                          W3(IT,IP,IE)),JJ=1,NX)
  170 CONTINUE
C
C     INITIALIZE SCALING FACTORS
C
      DO 180 II=1,NOPT
  180 SF(II)=1.0
C
C     ASSIGN DEFAULT VALUES
C
      IF(PLIM.LT.0.01) PLIM=0.01
      IF(TLIM.LT.0.01) TLIM=0.01
      IF(PLIM.GT.1.00) PLIM=1.00
      IF(TLIM.GT.1.00) TLIM=1.00
      IF(IDAT(2).LE.   0) IDAT(2)=10
      IF(IDAT(2).GT. 20) IDAT(2)=20
      IF(RDAT(1).LE.0.0) RDAT(1)=0.30
      IF(RDAT(2).LE.0.0) RDAT(2)=0.01
C
C     PRINT INPUT DATA
C
      WRITE(6,900) TITL
      WRITE(6,904) JOPT,JRUN,JPLT,NTST,KRSL,KNRM,
     *             POWR,PLIM,TLIM,NOPT,(KOPT(II),II=1,NOPT)
      WRITE(6,908) (PRP1(II),II=1,19)
      WRITE(6,912)
      DO 190 II=1,NTST
  190 WRITE(6,916) II,PRP2(II,4),(PRP2(II,JJ),JJ=1,3)
      WRITE(6,920) IDAT(2),RDAT(2),RDAT(1)
      IF(PRP1(6).EQ.0.0) WRITE(6,924)
      IF(PRP1(6).NE.0.0) WRITE(6,928)
      IF(IDAT(3).EQ.   0) WRITE(6,932)
      IF(IDAT(3).EQ.   1) WRITE(6,936)
      IF(IDAT(1).EQ.   0) WRITE(6,940)
```

```fortran
      IF(IDAT(1).EQ.  1) WRITE(6,944)
      WRITE(6,948)
      DO 210 II=1,NTST
      DO 200 JJ=1,NSEG(II)
      WRITE(6,952) II,JJ,LTYP(II,JJ,7),VALU(II,JJ,7),
     *              (LTYP(II,JJ,KK),KK=1,6),
     *              (VALU(II,JJ,KK),KK=1,6)
  200 CONTINUE
      WRITE(6,956)
  210 CONTINUE
C
C     ESTABLISH EXPERIMENTAL MAXIMA
C     IF MODEL CALIBRATION IS REQUIRED
C
      IF(JOPT.EQ.1) CALL FNDMAX(NTST,NSWT)
      RETURN
      END
C
C
C
      SUBROUTINE FNDMAX(NT,NS)
C
C     THIS SUBROUTINE ESTABLISHES THE MAXIMA
C     OF THE SPECIFIED EXPERIMENTAL OBSERVATIONS,
C     AS REQUIRED BY SUBROUTINE DATAIN.
C
      COMMON/BLKB/W1(6),W2(6,3),W3(6,3,20)
      COMMON/BLKD/NPLT(6),KPLT(6,3),NEXP(6,3),
     *            XV(6,3,20),YV(6,3,20),PINC(61,5)
      COMMON/BLKE/KRSL,POWR,XMXR(5),YMXR(5),XMXP(6,3),
     *            YMXP(6,3),PLIM,TLIM
C
C     ESTABLISH MAXIMA
C
      DO 100 II=1,5
      XMXR(II)=0.0
  100 YMXR(II)=0.0
      DO 120 II=1,NT
      DO 120 JJ=1,NPLT(II)
      KR=KPLT(II,JJ)
      XMXP(II,JJ)=ABS(XV(II,JJ,1))
      YMXP(II,JJ)=ABS(YV(II,JJ,1))
      DO 110 KK=2,NEXP(II,JJ)
      IF(XMXP(II,JJ).LT.ABS(XV(II,JJ,KK)))
     *      XMXP(II.JJ)=ABS(XV'II,JJ,KK))
  110 IF(YMXP(II,JJ .LT.ABS(    II,JJ,KK)))
     *      YMXP(II,__, ABS  V(II,JJ,KK))
      IF(XMXR(KR).LT. _ P(II,JJ)) XMXR(KR)=XMXP(II,JJ)
  120 IF(YMXR(KR).LT.YMXP(II,JJ)) YMXR(KR)=YMXP(II,JJ)
C
C     PRINT EXPERIMENTAL MAXIMA
C
      WRITE(6,130)
  130 FORMAT(//5X,'INPUT DATA:'/5X,'SPECIFIED EXPERIMENTAL ',
```

101

```
      *           'OBSERVATIONS AND WEIGHTING FACTORS:'/
      *           5X,'----------------------------------------',
      *           '--------------------'//
      *           5X,'TEST',3X,'PLOT',3X,'KIND',2X,'WF-TEST',
      *           2X,'WF-PLOT',2X,'#-PTS',2X,'YMAX(PLOT)',
      *           2X,'XMAX(PLOT)',2X,'YMAX(KIND)',2X,'XMAX(KIND)'/
      *           5X,'----',2(3X,'----'),
      *           2(2X,'-------'),2X,'-----',4(2X,'----------')/)
      DO 160 II=1,NT
      DO 140 JJ=1,NPLT(II)
      KR=KPLT(II,JJ)
  140 WRITE(6,150) II,JJ,KR,W1(II),W2(II,JJ),NEXP(II,JJ),
      *           YMXP(II,JJ),XMXP(II,JJ),YMXR(KR),XMXR(KR)
  150 FORMAT(2X,3(3X,I4),2F9.3,2X,I5,1P4E12.3)
  160 WRITE(6,170)
  170 FORMAT(20X)
C
C     PRINT SPECIAL EXPERIMENTAL WEIGHTINGS
C
      IF(NS.EQ.0) GO TO 210
      IS=0
      WRITE(6,180)
  180 FORMAT(//5X,'INPUT DATA:'/5X,'SPECIAL EXPERIMENTAL ',
      *        'WEIGHTINGS:'/5X,32('-')//5X,'PT-#',3X,'TEST',
      *        3X,'PLOT',2X,'POINT',6X,'WEIGHT'/2X,3(3X,'----'),
      *        2X,'-----',6X,'------'/)
      DO 190 II=1,NT
      DO 190 JJ=1,NPLT(II)
      DO 190 KK=1,NEXP(II,JJ)
      WT=W3(II,JJ,KK)
      IF(WT.NE.1.0) IS=IS+1
  190 IF(WT.NE.1.0) WRITE(6,200) IS,II,JJ,KK,WT
  200 FORMAT(2X,4(4X,I3),1PE12.3)
  210 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE EVAL(XOPT,RSDL,JCOD)
C
C     THIS SUBROUTINE PERFORMS A SINGLE-ELEMENT,
C     INCREMENTAL-ITERATIVE FINITE ELEMENT ANALYSIS
C     FOR HOMOGENEOUS LOADING CONDITIONS.  FOR USE
C     WITH MATERIAL MODELS SUCH AS THE BOUNDING
C     SURFACE SOIL PLASTICITY MODEL.
C
C     WRITTEN BY L.R. HERRMANN,
C     RECODED BY J.S. DE NATALE,
C     DEPARTMENT OF CIVIL ENGINEERING,
C     UNIVERSITY OF CALIFORNIA, DAVIS.
C     LAST REVISED: MAY 1982.
C
      COMMON/BLK7/SFUN,SF(17),XL(17),XU(17)
```

```
      COMMON/BLKA/NTST,JOPT,JPLT,NOPT,KOPT(17)
      COMMON/BLKB/W1(6),W2(6,3),W3(6,3,20)
      COMMON/BLKC/PRP1(19),PRP2(6,4),IDAT(3),RDAT(2),
     *            NSEG(6),LTYP(6,4,7),VALU(6,4,7)
      COMMON/BLKD/NPLT(6),KPLT(6,3),NEXP(6,3),
     *            XV(6,3,20),YV(6,3,20),PINC(61,5)
      COMMON/BLKE/KRSL,POWR,XMXR(5),YMXR(5),XMXP(6,3),
     *            YMXP(6,3),PLIM,TLIM
      DIMENSION   ILOD(6),VLOD(6),DLOD(6),SIGB(6),EPSB(6),DSIG(6),
     *            DEPS(6),STOR(6),PROP(21),C(6,6),S(7,7),R(7),RP(6)
      DIMENSION   XOPT(17)
C
C     FORMAT STATEMENTS
C
  900 FORMAT(1H1,2X,'OUTPUT DATA:'/3X,'MODEL PREDICTIONS ',
     *       'CORRESPONDING TO TEST DATA SET #',I2,' ARE AS FOLLOWS:'
     *       /3X,17('-----')
     *       ///3X,'N',4X,'EPS-X',4X,'EPS-Y',4X,'EPS-Z',3X,'GAM-XY',
     *       3X,'GAM-XZ',3X,'GAM-YZ',5X,'SIG-X',5X,'SIG-Y',5X,'SIG-Z',
     *       4X,'TAU-XY',4X,'TAU-XZ',4X,'TAU-YZ',6X,'U',3X,'#-IT'/
     *       3X,'-',3(4X,'-----'),3(3X,'------'),3(5X,'-----'),
     *       3(4X,'------'),5X,'---',2X,'----'/)
  904 FORMAT(//3X,'CONVERGENC: ID NOT OCCUR FOR INCREMENT',I3,
     *       ' ---   ERSIG=',E9.3,' AND EREPS=',E9.3//)
  908 FORMAT(1X,I3,1P6E9.1,7E10.2,1X,I3)
  912 FORMAT(/)
  916 FORMAT(1H1,2X,'OUTPUT DATA:'/3X,'MODEL PREDICTIONS ',
     *       '-- IN TERMS OF THE KEY RESPONSE'/3X,'PARAMETERS -- ',
     *       'CORRESPONDING TO TEST DATA SET #',I2,' ARE AS FOLLOWS:'
     *       /3X,16('-----')//16X,'N',5X,'EPS-1',9X,'Q',9X,'P',9X,'U',
     *       5X,'E-VOL'/16X,'-',5(5X,'-----')/)
  920 FORMAT(12X,I5,5F10.2)
C
C     INDIVIDUAL TEST LOOP
C
      RSDL=0.0
      WTST=0.0
      DO 700 L1=1,NTST
C
C     INITIALIZE ALL PARAMETERS
C
      NO=0
      XX=1.00E+25
      DU1T=0.0
      XNRME1=0.0
      XNRMS1=0.0
      DO 100 II=1,6
      EPSB(II)=0.0
      DEPS(II)=0.0
      SIGB(II)=0.0
  100 DSIG(II)=0.0
      U=0.0
      DELU=0.0
C
```

103

```
C     ASSIGN VALUES TO THE TEST UNDER CONSIDERATION
C
      DO 105 II=1,19
  105 PROP(II)=PRP1(II)
      PROP(20)=PRP2(L1,1)
      PROP(21)=PRP2(L1,2)
      PATM=PROP(8)
      PCON=PRP2(L1,3)
      KIND=IDAT(1)
      CNFR=RDAT(1)
      ITMX=IDAT(2)
      ERMX=RDAT(2)
      LARG=IDAT(3)
      DO 110 II=1,3
  110 SIGB(II)=PCON
      DO 115 II=1,5
  115 PINC(1,II)=0.0
      PINC(1, 2)=PCON
C
C     ACCOUNT FOR CHANGES IN THOSE
C     PARAMETERS WHOSE OPTIMAL VALUES ARE BEING SOUGHT
C
      IF(JOPT.EQ.0) GO TO 125
      DO 120 II=1,NOPT
  120 PROP(KOPT(II))=XL(II)+(XU(II)-XL(II))*
     *                  (SIN(XOPT(II)/SF(II)))**2
  125 CONTINUE
C
C     SCALE PARAMETERS
C
      PROP( 3)=PROP( 3)/3.0/SQRT(3.0)
      PROP( 7)=PROP( 7)*3.0
      PROP(21)=PROP(21)*3.0
C
C     LOADING HISTORY SEGMENT LOOP
C
      IF(JCOD.EQ.1) WRITE(6,900) L1
      DO 600 L2=1,NSEG(L1)
      DO 130 II=1,6
      ILOD(II)=LTYP(L1,L2,II)
  130 VLOD(II)=VALU(L1,L2,II)
      NINC=LTYP(L1,L2,7)
      SRAT=VALU(L1,L2,7)
C
C     DETERMINE FIRST INCREMENTS
C
      D1=FLOAT(NINC)
      D1=1.0/D1
      IF(SRAT.NE.1.0) D1=(1.0-SRAT)/(1.0-SRAT**NINC)
      DO 135 II=1,6
      D2=VLOD(II)-SIGB(II)
      IF(ILOD(II).EQ.1) D2=VLOD(II)-EPSB(II)
  135 DLOD(II)=D1*D2
C
```

104

```
C      CHANGE THE SIGN OF THE STRAIN ESTIMATE AT THE
C      BEGINNING OF THE NEXT SEGMENT IF THERE HAS BEEN
C      UNSTABLE BEHAVIOR AT THE END OF THE PREVIOUS ONE
C
       DELU= 0.01*DELU
       DO 140 II=1,6
       DSIG(II)= 0.01*DSIG(II)
  140 DEPS(II)=-0.01*DEPS(II)
C
C      INCREMENT LOOP
C
       DO 500 INCR=1,NINC
C
C      ITERATION LOOP (SUCCESSIVE APPROXIMATION)
C
       DO 400 ITNO=1,ITMX
C
C      ESTABLISH INCREMENTAL PROPERTIES
C
       NJ=NO+1
       K7=ITNO
       CALL CLAY(3,NJ,K7,PROP,STOR,SIGB,EPSB,
      *          DSIG,DEPS,C,U,DELU,GAMA,KIND,LARG)
C
C      FORM AND MODIFY STIFFNESS MATRICES
C
       DO 200 II=1,3
       S(7,II)=GAMA
       S(7,II+3)=0.0
       S(II,7)=1.0
  200 S(II+3,7)=0.0
       S(7,7)=-1.0
       R(7)=0.0
       DO 210 II=1,6
       DO 205 JJ=1,6
  205 S(II,JJ)=C(II,JJ)
       R(II)=DLOD(II)
       IF(ILOD(II).EQ.0) GO TO 210
       S(II,II)=XX
       R(II)=DLOD(II)*XX
  210 CONTINUE
C
C      SOLVE FOR STRAIN INCREMENT
C
       NN=7-KIND
       DO 230 II=1,NN
       D2=1.0/S(II,II)
       R(II)=R(II)*D2
       DO 215 JJ=II,NN
  215 S(II,JJ)=S(II,JJ)*D2
       IF(II.EQ.NN) GO TO 230
       IL=II+1
       DO 225 JJ=IL,NN
       D2=-S(JJ,II)
```

105

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      DO 220 KK=II,NN
  220 S(JJ,KK)=S(JJ,KK)+D2*S(II,KK)
  225 R(JJ)=R(JJ)+D2*R(II)
  230 CONTINUE
      IC=NN
      DO 235 II=2,NN
      IC=IC-1
      IL=IC+1
      DO 235 JJ=IL,NN
  235 R(IC)=R(IC)-S(IC,JJ)*R(JJ)
C
C     COMPUTE STRESS INCREMENT
C
      DO 245 II=1,6
      D2=0.0
      IF(II.LT.4) D2=R(7)
      DO 240 JJ=1,6
  240 D2=D2+C(II,JJ)*R(JJ)
  245 RP(II)=D2
C
C     COMPUTE ERROR NORMS
C
      ESIG=0.0
      EEPS=0.0
      D1=0.0
      D2=0.0
      DO 250 II=1,6
      ESIG=ESIG+ABS(RP(II)-DSIG(II))
      EEPS=EEPS+ABS(R (II)-DEPS(II))
      D1=D1+ABS(RP(II))
  250 D2=D2+ABS(R (II))
      DU1P=D1
      IF(DU1P.LT.DU1T) DU1P=DU1T
      ESIG=ESIG/DU1P
      EEPS=EEPS/D2
C
C     CHECK FOR CONVERGENCE
C
      IF(ESIG.LT.ERMX.AND.EEPS.LT.ERMX) GO TO 405
C
C     APPLY AITKEN'S NORM ACCELERATION
C
      CNFS=1.0
      CNFE=1.0
      IF(ITNO.EQ.1.OR.((-1)**ITNO).GT.0) GO TO 255
      CALL AITKEN(XNRMS2,XNRMS1,D1,CNFS,CNFR)
      CALL AITKEN(XNRME2,XNRME1,D2,CNFE,CNFR)
  255 D1=0.0
      D2=0.0
      DO 260 II=1,6
      DSIG(II)=CNFS*RP(II)+(1.0-CNFS)*DSIG(II)
      DEPS(II)=CNFE*R (II)+(1.0-CNFE)*DEPS(II)
      D1=D1+ABS(DSIG(II))
  260 D2=D2+ABS(DEPS(II))
```

106

```
              IF(KIND.EQ.0) DELU=CNFS*R(7)+(1.0-CNFS)*DELU
              XNRMS2=XNRMS1
              XNRME2=XNRME1
              XNRMS1=D1
              XNRME1=D2
          400 CONTINUE
              IF(JCOD.EQ.1) WRITE(6,904) NO+1,ESIG,EEPS
              GO TO 605
      C
      C     UPDATE TOTAL VALUES
      C
          405 NO=NO+1
              DU1T=0.0
              DO 410 II=1,6
              DLOD(II)=DLOD(II)*SRAT
              DSIG(II)=RP(II)
              DEPS(II)=R(II)
              SIGB(II)=SIGB(II)+DSIG(II)
              EPSB(II)=EPSB(II)+DEPS(II)
          410 DU1T=DU1T+0.1*ABS(SIGB(II))
              IF(KIND.EQ.0) DELU=R(7)
              U=U+DELU
      C
      C     STORE INCREMENTAL VALUES FOR FUTURE PLOTTING
      C
              NJ=NO+1
              PINC(NJ,1)=(SIGB(3)-SIGB(1))
              PINC(NJ,2)=(SIGB(1)+SIGB(2)+SIGB(3)) /3.0-U
              PINC(NJ,3)= U
              PINC(NJ,4)=(EPSB(1)+EPSB(2)+EPSB(3))*100.00
              PINC(NJ,5)= EPSB(3)*100.00
      C
      C     PRINT INCREMENTAL VALUES
      C
              IF(JCOD.EQ.1) WRITE(6,908) NO,(EPSB(II),II=1,6),
             *                          (SIGB(II),II=1,6),U,ITNO
          500 CONTINUE
              IF(JCOD.EQ.1) WRITE(6,912)
          600 CONTINUE
          605 CONTINUE
              IF(JCOD.EQ.0) GO TO 615
              WRITE(6,916) L1
              DO 610 II=2,NJ
              JJ=II-1
          610 WRITE(6,920) JJ,PINC(II,5),(PINC(II,KK),KK=1,4)
          615 CONTINUE
      C
      C     COMPUTE RESIDUALS IF MODEL CALIBRATION IS REQUIRED
      C
              ITST=L1
              IF(JCOD.EQ.1) GO TO 620
              CALL RSDUAL(NJ,ITST,RTST)
              RSDL=RSDL+RTST*W1(L1)
              WTST=WTST+W1(L1)
```

107

```
  620 CONTINUE
C
C      PLOT MODEL PREDICTIONS IF PLOTTING IS REQUIRED
C
       IF(JCOD.EQ.1.AND.JPLT.EQ.1) CALL PLTCHK(NJ,ITST)
  700 CONTINUE
       IF(WTST.NE.0.0) RSDL=RSDL/WTST
       RETURN
       END
C
C
C
       SUBROUTINE AITKEN(X2,X1,XX,XC,XL)
C
C      THIS SUBROUTINE COMPUTES
C      THE AITKEN'S CONVERGENCE FACTORS.
C
       XC=1.0
       DX=2.0*X1-X2-XX
       IF(DX.EQ.0.0) RETURN
       XC=(X - ?)/DX
       IF(XC.LT.XL) XC=XL
       IF(XC.GT.(1.0/XL)) XC=1.0/XL
       RETURN
       END
C
C
C
       SUBROUTINE PLTCHK(NP,IT)
C
C      THIS SUBROUTINE SERVES AS THE DRIVING PROGRAM
C      FOR THE ASSOCIATED PLOTTING SUBROUTINES JSDPLT,
C      LSHIFT AND BORDER.
C
C      WRITTEN BY J.S. DE NATALE,
C      DEPARTMENT OF CIVIL ENGINEERING,
C      UNIVERSITY OF CALIFORNIA, DAVIS.
C      VERSION I: MAY 1982.
C
       COMMON/BLKD/NPLT(6),KPLT(6,3),NEXP(6,3),
      *            XV(6,3,20),YV(6,3,20),PINC(61,5)
       DIMENSION   IPLT(10)
C
C      CHECK WHETHER PLOTTING IS REQUIRED (NOTE:
C      ADDITIONAL RELATION TYPES MAY BE INCLUDED AS NEEDED)
C
       IP=0
       DO 100 II=1,10
  100 IPLT(II)=0
       DO 110 II=1,NPLT(IT)
  110 IPLT(KPLT(IT,II))=1
       IF(IPLT( 1).EQ.1)
      *          CALL JSDPLT(IT,IP,NP, 1, 2,' Q ',' P ')
       IF(IPLT( 2).EQ.1)
```

108

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      *           CALL JSDPLT(IT,IP,NP, 1, 5,'  Q  ','EPS-Z')
            IF(IPLT( 3).EQ.1)
      *           CALL JSDPLT(IT,IP,NP, 2, 5,'  P  ','EPS-Z')
            IF(IPLT( 4).EQ.1)
      *           CALL JSDPLT(IT,IP,NP, 3, 5,'  U  ','EPS-Z')
            IF(IPLT( 5).EQ.1)
      *           CALL JSDPLT(IT,IP,NP, 4, 5,'EPS-V','EPS-Z')
            RETURN
            END
C
C
C
C
            SUBROUTINE JSDPLT(IT,IP,NP,IY,IX,TY,TX)
C
C     THIS SUBROUTINE DIRECTS THE PLOTTING OF THE SPECIFIED
C     EXPERIMENTAL OBSERVATIONS AND THEORETICAL MODEL PREDICTIONS.
C
C     WRITTEN BY J.S. DE NATALE,
C     DEPARTMENT OF CIVIL ENGINEERING,
C     UNIVERSITY OF CALIFORNIA, DAVIS.
C     VERSION I: MAY 1982.
C
      COMMON/BLKD/NPLT(6),KPLT(6,3),NEXP(6,3),
      *            XV(6,3,20),YV(6,3,20),PINC(61,5)
      DIMENSION XLAB(11),YLAB(11),ICPX(61),ICPY(61),ICEX(20),ICEY(20)
      DIMENSION FSPA(101),FORM(4),FSYM(2)
      DOUBLE PRECISION TX,TY
      DATA FSPA/'     ',' 1X,',' 2X,',' 3X,',' 4X,',' 5X,',' 6X,',' 7X,'
      *          ,' 8X,',' 9X,','10X,','11X,','12X,','13X,','14X,','15X,'
      *          ,'16X,','17X,','18X,','19X,','20X,','21X,','22X,','23X,'
      *          ,'24X,','25X,','26X,','27X,','28X,','29X,','30X,','31X,'
      *          ,'32X,','33X,','34X,','35X,','36X,','37X,','38X,','39X,'
      *          ,'40X,','41X,','42X,','43X,','44X,','45X,','46X,','47X,'
      *          ,'48X,','49X,','50X,','51X,','52X,','53X,','54X,','55X,'
      *          ,'56X,','57X,','58X,','59X,','60X,','61X,','62X,','63X,'
      *          ,'64X,','65X,','66X,','67X,','68X,','69X,','70X,','71X,'
      *          ,'72X,','73X,','74X,','75X,','76X,','77X,','78X,','79X,'
      *          ,'80X,','81X,','82X,','83X,','84X,','85X,','86X,','87X,'
      *          ,'88X,','89X,','90X,','91X,','92X,','93X,','94X,','95X,'
      *          ,'96X,','97X,','98X,','99X,','99X,'/
      DATA FORM/'(1H+','19X,','     ','     '/
      DATA FSYM/'1H*)','1H#)'/
      IP=IP+1
      NE=NEXP(IT,IP)
C
C     ESTABLISH MINIMUM AND MAXIMUM AXES VALUES
C
      XMIN=PINC(1,IX)
      XMAX=PINC(1,IX)
      YMIN=PINC(1,IY)
      YMAX=PINC(1,IY)
      DO 100 II=2,NP
      IF(XMIN.GT.PINC(II,IX)) XMIN=PINC(II,IX)
      IF(XMAX.LT.PINC(II,IX)) XMAX=PINC(II,IX)
```

109

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       IF(YMIN.GT.PINC(II,IY)) YMIN=PINC(II,IY)
  100  IF(YMAX.LT.PINC(II,IY)) YMAX=PINC(II,IY)
       IF(NE.EQ.0) GO TO 120
       DO 110 II=1,NE
       IF(XMIN.GT.XV(IT,IP,II)) XMIN=XV(IT,IP,II)
       IF(XMAX.LT.XV(IT,IP,II)) XMAX=XV(IT,IP,II)
       IF(YMIN.GT.YV(IT,IP,II)) YMIN=YV(IT,IP,II)
  110  IF(YMAX.LT.YV(IT,IP,II)) YMAX=YV(IT,IP,II)
  120  CONTINUE
C
C      ESTABLISH ADJUSTED MAXIMUM AND MINIMUM AXES VALUES
C
       CALL LSHIFT(XMIN,XMAX)
       CALL LSHIFT(YMIN,YMAX)
C
C      ESTABLISH AXES LABLES
C
       XDST=XMAX-XMIN
       YDST=YMAX-YMIN
       XINC=XDST/10.0
       YINC=YDST/10.0
       XLAB( 1)=XMIN
       XLAB(11)=XMAX
       YLAB( 1)=YMIN
       YLAB(11)=YMAX
       DO 130 II=2,10
       AA=FLOAT(II)
       AA=FLOAT(II-1)
       XLAB(II)=XMIN+XINC*AA
  130  YLAB(II)=YMIN+YINC*AA
C
C      ESTABLISH DATA COORDINATES
C
       DO 140 II=1,NP
       A1=((PINC(II,IX)- XMIN)/XDST)*100.0
       IF(A1.LT.0.0) A1=0.0
       A2=AINT(A1)
       IF(A1-A2.GT.0.5) A2=A2+1.0
       ICPX(II)=IFIX(A2)
       A1=((PINC(II,IY)- YMIN)/YDST)* 50.0
       IF(A1.LT.0.0) A1=0.0
       A2=AINT(A1)
       IF(A1-A2.GT.0.5) A2=A2+1.0
  140  ICPY(II)=IFIX(A2)
       IF(NE.EQ.0) GO TO 160
       DO 150 II=1,NE
       A1=((XV(IT,IP,II)-XMIN)/XDST)*100.0
       IF(A1.LT.0.0) A1=0.0
       A2=AINT(A1)
       IF(A1-A2.GT.0.5) A2=A2+1.0
       ICEX(II)=IFIX(A2)
       A1=((YV(IT,IP,II)-YMIN)/YDST)* 50.0
       IF(A1.LT.0.0) A1=0.0
       A2=AINT(A1)
```

```
          IF(A1-A2.GT.0.5) A2=A2+1.0
  150 ICEY(II)=IFIX(A2)
C
C     PRINT PLOT
C
  160 WRITE(6,170)
  170 FORMAT(1H1/)
          II=1
          JJ=0
          DO 210 I2=1,51
C
C     PRINT HEADINGS AND AXES LABLES
C
          K7=I2
          CALL BORDER(II,JJ,K7,XLAB,YLAB,TX,TY)
C
C     PRINT DATA POINTS
C
          I3=51-I2
          DO 180 J2=1,NP
          IF(ICPY(J2).EQ.I3) FORM(4)=FSYM(1)
          IF(ICPY(J2).EQ.I3) FORM(3)=FSPA(ICPX(J2)+1)
  180 IF(ICPY(J2).EQ.I3) WRITE(6,FORM)
          IF(NE.EQ.0) GO TO 200
          DO 190 J2=1,NE
          IF(ICEY(J2).EQ.I3) FORM(4)=FSYM(2)
          IF(ICEY(J2).EQ.I3) FORM(3)=FSPA(ICEX(J2)+1)
  190 IF(ICEY(J2).EQ.I3) WRITE(6,FORM)
  200 CONTINUE
  210 CONTINUE
          WRITE(6,220) (XLAB(MM),MM=1,11)
  220 FORMAT(/16X,11(F8.4,2X))
          WRITE(6,230) TX
  230 FORMAT(/20X,48X,A5)
          RETURN
          END
C
C
C
          SUBROUTINE LSHIFT(PMIN,PMAX)
C
C     THIS SUBROUTINE ADJUSTS THE MINIMUM
C     AND MAXIMUM AXES VALUES, AS REQUIRED
C     BY SUBROUTINE JSDPLT.
C
          DX=(PMAX-PMIN)/10.0
          XL=PMIN-DX
          XU=PMAX+DX
          D0=ABS(PMIN)
          D1=ABS(PMAX)
          IF(D1.LT.D0) D1=D0
          NN=21
          DO 100 JJ=1,40
          NN=NN-1
```

```
      C1=10.0**NN
      IF(D1.GE.C1) GO TO 110
  100 CONTINUE
  110 D1=PMAX/C1
      II=IFIX(D1)
      DU=FLOAT(II)*C1
      IF(DU.LT.PMAX) II=II+1
      DU=FLOAT(II)*C1
      IF(DU.LE.XU) GO TO 120
      C1=C1/10.0
      GO TO 110
  120 D1=PMIN/C1
      II=IFIX(D1)
      DL=FLOAT(II)*C1
      IF(DL.GT.PMIN) II=II-1
      DL=FLOAT(II)*C1
      IF(DL.GE.XL) GO TO 130
      C1=C1/10.0
      GO TO 110
  130 PMIN=DL
      PMAX=DU
      RETURN
      END
C
C
C
      SUBROUTINE BORDER(II,JJ,I2,XLAB,YLAB,XT,YT)
C
C     THIS SUBROUTINE PRODUCES THE PLOT HEADINGS
C     AND AXES LABLES, AS REQUIRED BY SUBROUTINE JSDPLT.
C
      DIMENSION XLAB(11),YLAB(11)
      DOUBLE PRECISION XT,YT
      KK=0
      IF(I2.EQ.1.OR.I2-II.EQ.5) KK=1
      IF(KK.EQ.1)  II=I2
      IF(KK.EQ.1)  JJ=JJ+1
      LL=12-JJ
      IF(I2.EQ. 1) WRITE(6,110) YLAB(LL)
      IF(I2.EQ. 1) GO TO 100
      IF(I2.EQ.26) WRITE(6,120) YT,YLAB(LL)
      IF(I2.EQ.26) GO TO 100
      IF(I2.EQ.51) WRITE(6,110) YLAB(LL)
      IF(I2.EQ.51) GO TO 100
      IF(KK.EQ. 1) WRITE(6,130) YLAB(LL)
      IF(KK.EQ. 0) WRITE(6,140)
  100 CONTINUE
  110 FORMAT(1X,5X,2X,F10.5,2X,'I',10('---------I'))
  120 FORMAT(1X,A5,2X,F10.5,2X,'+-',97X,'-+')
  130 FORMAT(1X,5X,2X,F10.5,2X,'+-',97X,'-+')
  140 FORMAT(1X,5X,2X,   10X,2X,'I ',97X,' I')
      RETURN
      END
C
```

112

```
C
C
        SUBROUTINE RSDUAL(NP,IT,RTST)
C
C       THIS SUBROUTINE COMPUTES THE WEIGHTED
C       ABSOLUTE OR SQUARED RESIDUAL BETWEEN THE
C       EXPERIMENTAL OBSERVATIONS AND THE CONSTITUTIVE
C       MODEL PREDICTIONS.
C
C       WRITTEN BY J.S. DE NATALE,
C       DEPARTMENT OF CIVIL ENGINEERING,
C       UNIVERSITY OF CALIFORNIA, DAVIS.
C       VERSION I: MAY 1982
C
        COMMON/BLKB/W1(6),W2(6,3),W3(6,3,20)
        COMMON/BLKD/NPLT(6),KPLT(6,3),NEXP(6,3),
     *              XV(6,3,20),YV(6,3,20),PINC(61,5)
        COMMON/BLKE/KRSL,POWR,XMXR(5),YMXR(5),XMXP(6,3),
     *              YMXP(6,3),PLIM,TLIM
C
C       INDIVIDUAL PLOT LOOP
C
        RTST=0.0
        WPLT=0.0
        DO 400 IP=1,NPLT(IT)
C
C       IDENTIFY RELEVANT MAXIMA
C
        GO TO (100,102,104,106,108), KPLT(IT,IP)
  100 IX=2
        IY=1
        XALL=XMXR(1)
        YALL=YMXR(1)
        GO TO 120
  102 IX=5
        IY=1
        XALL=XMXR(2)
        YALL=YMXR(2)
        GO TO 120
  104 IX=5
        IY=2
        XALL=XMXR(3)
        YALL=YMXR(3)
        GO TO 120
  106 IX=5
        IY=3
        XALL=XMXR(4)
        YALL=YMXR(4)
        GO TO 120
  108 IX=5
        IY=4
        XALL=XMXR(5)
        YALL=YMXR(5)
  120 CONTINUE
```

113

```fortran
C
C        COMPUTE AND SUM WEIGHTED RESIDUALS
C
         IF(KRSL.NE.0) LL=1
         IF(KRSL.EQ.0) LL=2
         RPLT=0.0
         WPNT=0.0
         XMAX=XMXP(IT,IP)
         YMAX=YMXP(IT,IP)
C
C        CHECK FOR ABSOLUTE OR RELATIVE TEST SCALING
C
         XMXT=XMAX
         YMXT=YMAX
         IF((YMAX/YALL).LT.TLIM) YMXT=YALL*TLIM
C
C        INDIVIDUAL EXPERIMENTAL OBSERVATION LOOP
C
         DO 300 IE=1,NEXP(IT,IP)
         RPNT=0.0
C
C        MODEL PREDICTIONS LOOP (IN CALLED SUBROUTINE)
C
         IMIN=0
         IPLT=IP
         IEXP=IE
         IF(KRSL.NE.0) CALL VRTICL(IT,IPLT,IEXP,NP,IX,IY,
     *                              LL,YE,DR,IMIN)
         IF(KRSL.EQ.0) CALL EUCLID(IT,IPLT,IEXP,NP,IX,IY,
     *                              LL,YE,DR,IMIN,XMXT,YMXT)
C
C        CHECK FOR ABSOLUTE OR RELATIVE POINT SCALING
C
         IF(IMIN.EQ.0) GO TO 200
         YE=ABS(YE)
         YMXV=YE
         IF((YE/YMAX).LT.PLIM) YMXV=YMAX*PLIM
         RPNT=((ABS(DR)/YMXV)*(YMAX/YMXT))**POWR
         RPLT=RPLT+RPNT*W3(IT,IP,IE)
         WPNT=WPNT+W3(IT,IP,IE)
  200 CONTINUE
  300 CONTINUE
         RPLT=RPLT/WPNT
         RTST=RTST+RPLT*W2(IT,IP)
         WPLT=WPLT+W2(IT,IP)
  400 CONTINUE
         RTST=RTST/WPLT
         RETURN
         END
C
C
C
         SUBROUTINE EUCLID(IT,IP,IE,NP,IX,IY,LL,YE,DR,
     *                     IMIN,XMXT,YMXT)
```

114

```fortran
C
C      THIS SUBROUTINE COMPUTES THE MINIMUM EUCLIDEAN
C      DISTANCE BETWEEN THE EXPERIMENTAL OBSERVATION AND
C      THE MODEL PREDICTION "CURVE".
C
C      WRITTEN BY J.S. DE NATALE,
C      DEPARTMENT OF CIVIL ENGINEERING,
C      UNIVERSITY OF CALIFORNIA, DAVIS.
C      VERSION I: MAY 1982.
C
       COMMON/BLKD/NPLT(6),KPLT(6,3),NEXP(6,3),
      *           XV(6,3,20),YV(6,3,20),PINC(61,5)
C
C      DETERMINE THE PREDICTION POINT WHICH IS NEAREST
C      TO THE EXPERIMENTAL OBSERVATION IN A SUITABLY SCALED
C      X-Y SPACE
C
       ML=0
       DR=1.00E+02
       XE=XV(IT,IP,IE)/XMXT
       YE=YV(IT,IP,IE)/YMXT
       DO 100 KL=LL,NP-1
       XP=PINC(KL,IX)/XMXT
       YP=PINC(KL,IY)/YMXT
       DV=SQRT((XE-XP)**2+(YE-YP)**2)
       IF(DR.GT.DV) ML=KL
  100  IF(DR.GT.DV) DR=DV
       IF(ML.EQ. 0) GO TO 120
C
C      SEARCH ADJACENT SEGMENTS FOR AN EVEN SMALLER DISTANCE
C
       NL=ML
       DO 110 MM=1,2
       K1=NL-1
       K2=NL
       X1=PINC(K1,IX)/XMXT
       Y1=PINC(K1,IY)/YMXT
       X2=PINC(K2,IX)/XMXT
       Y2=PINC(K2,IY)/YMXT
       DX=X2-X1
       DY=Y2-Y1
       BETA=((XE-X1)*DX+(YE-Y1)*DY)/(DX**2+DY**2)
       IF(BETA.LT.0.0) BETA=0.0
       IF(BETA.GT.1.0) BETA=1.0
       XM=X1+BETA*DX
       YM=Y1+BETA*DY
       DV=SQRT((XE-XM)**2+(YE-YM)**2)
       IF(DR.GT.DV) DR=DV
  110  NL=NL+1
       LL=ML
       IMIN=1
  120  DR=DR*YMXT
       YE=YE*YMXT
       RETURN
```

```
          END
C
C
C
          SUBROUTINE VRTICL(IT,IP,IE,NP,IX,IY,LL,YE,DR,IMIN)
C
C         THIS SUBROUTINE COMPUTES THE MINIMUM VERTICAL
C         DISTANCE BETWEEN THE EXPERIMENTAL OBSERVATION AND THE
C         MODEL PREDICTION "CURVE".
C
C         WRITTEN BY J.S. DE NATALE,
C         DEPARTMENT OF CIVIL ENGINEERING,
C         UNIVERSITY OF CALIFORNIA, DAVIS.
C         VERSION I: MAY 1982.
C
          COMMON/BLKD/NPLT(6),KPLT(6,3),NEXP(6,3),
     *               XV(6,3,20),YV(6,3,20),PINC(61,5)
C
C         DETERMINE THE PREDICTION POINTS WHICH MOST CLOSELY
C         BOUND (WRT THE X-VALUE) THE EXPERIMENTAL OBSERVATION
C
          XE=XV(IT,IP,IE)
          YE=YV(IT,IP,IE)
          DO 100 KL=LL,NP-1
          KU=KL+1
          XL=PINC(KL,IX)
          YL=PINC(KL,IY)
          XU=PINC(KU,IX)
          YU=PINC(KU,IY)
          IF(XE.EQ.XL) GO TO 110
          IF(XE.EQ.XU) GO TO 120
          IF(XE.LT.XL.AND.XE.GT.XU) GO TO 130
  100 IF(XE.GT.XL.AND.XE.LT.XU) GO TO 130
          GO TO 150
  110 YP=YL
          GO TO 140
  120 YP=YU
          GO TO 140
  130 BETA=(XE-XL)/(XU-XL)
          YP=YL+  BETA*(YU-YL)
  140 DR=ABS(YE-YP)
          LL=KL
          IMIN=1
  150 CONTINUE
          RETURN
          END
C
C
C
          SUBROUTINE BSMOPT(XX)
C
C         THIS SUBROUTINE DIRECTS THE CALIBRATION OF THE
C         BOUNDING SURFACE PLASTICITY MODEL.  THE OPTIMAL
C         VALUES OF THE MATERIAL AND MODEL PARAMETERS ARE
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116

```
C       SELECTED BY MINIMIZING THE WEIGHTED RESIDUAL BETWEEN
C       THE EXPERIMENTAL OBSERVATIONS AND THE MODEL PREDICTIONS.
C
C       WRITTEN BY J.S. DE NATALE,
C       DEPARTMENT OF CIVIL ENGINEERING,
C       UNIVERSITY OF CALIFORNIA, DAVIS.
C       VERSION I: MAY 1982.
C
        COMMON/BLK1/ETAF,ETAX,DMIN,FDER
        COMMON/BLK2/NDIM,NFMX,C1,C2,C3
        COMMON/BLK3/NI,NF,NG,F1,F2,S1,S2,X1(17),X2(17)
        COMMON/BLK4/EPFA,EPSA,ICON,JCON,IPRN,ICUB,XE(17)
        COMMON/BLK5/D1(17),D2(17),G1(17),G2(17),DG(17),DX(17),
     *              H1(17,17),H2(17,17)
        COMMON/BLK6/EPFL,EPXL,NFLX
        COMMON/BLK7/SFUN,SF(17),XL(17),XU(17)
        DIMENSION   XX(17)
C
C       READ CALIBRATION SPECIFICATIONS
C
        CALL CALDAT
C
C       INITIALIZE GLOBAL SEARCH
C
        DO 100 II=1,NDIM
        TX=X2(II)
        TD=X2(II)+DX(II)
        X2(II)=ASIN(SQRT((TX-XL(II))/(XU(II)-XL(II))))
        DX(II)=ASIN(SQRT((TD-XL(II))/(XU(II)-XL(II))))-X2(II)
        X2(II)=X2(II)*SF(II)
  100   DX(II)=DX(II)*SF(II)
        NF=0
        NG=0
        NI=0
        F2=FFUN(X2)
        NF=NF+1
        DO 110 II=1,NDIM
        IG=II
  110   G2(IG)=GFUN(IG,0,NDIM)
        NG=NG+1
        CALL MVPROD(H2,G2,D2,NDIM)
        S2=DTPROD(G2,D2,NDIM)
        IF(IPRN.GE.1) CALL PRNOUT(1,NDIM)
C
C       PERFORM LINE SEARCH
C
  200   CALL SEARCH
C
C       IF NO MINIMUM HAS BEEN FOUND, UPDATE THE
C       INVERSE HESSIAN, COMPUTE A NEW SEARCH DIRECTION.
C       AND PROCEED WITH THE NEXT ITERATION
C
        IF(JCON.EQ.1) GO TO 300
        CALL BFGSUP(S2,IHES,NI,NDIM)
```

```fortran
      JCON=0
      IF(ICON.EQ.3) CALL CONCHK(DF,S2,DX,NDIM)
      IF(JCON.EQ.1) CALL PRNOUT(1,NDIM)
      IF(JCON.EQ.1) CALL PRNOUT(5,NDIM)
      IF(JCON.EQ.1) GO TO 300
      GO TO 200
C
C     TERMINATE GLOBAL SEARCH
C
  300 CONTINUE
      DO 310 II=1,NDIM
  310 XX(II)=X2(II)
      RETURN
      END
C
C
C

      SUBROUTINE CALDAT
C
C     THIS SUBROUTINE READS IN ALL ADDITIONAL
C     INFORMATION REQUIRED TO DIRECT THE MODEL
C     CALIBRATION ALGORITHM.
C
C     WRITTEN BY J.S. DE NATALE,
C     DEPARTMENT OF CIVIL ENGINEERING,
C     UNIVERSITY OF CALIFORNIA, DAVIS.
C     VERSION I: MAY 1982.
C
      COMMON/BLKA/NTST,JOPT,JPLT,NOPT,KOPT(17)
      COMMON/BLKC/PRP1(19),PRP2(6,4),IDAT(3),RDAT(2),
     *            NSEG(6),LTYP(6,4,7),VALU(6,4,7)
      COMMON/BLK1/ETAF,ETAX,DMIN,FDER
      COMMON/BLK2/NDIM,NFMX,C1,C2,C3
      COMMON/BLK3/NI,NF,NG,F1,F2,S1,S2,X1(17),X2(17)
      COMMON/BLK4/EPFA,EPSA,ICON,JCON,IPRN,ICUB,XE(17)
      COMMON/BLK5/D1(17),D2(17),G1(17),G2(17),DG(17),DX(17),
     *            H1(17,17),H2(17,17)
      COMMON/BLK6/EPFL,EPXL,NFLX
      COMMON/BLK7/SFUN,SF(17),XL(17),XU(17)
C
C     FORMAT STATEMENTS
C
  800 FORMAT(16I5)
  804 FORMAT(7E10.3,2I5)
  808 FORMAT(8E10.3)
  812 FORMAT(4(2I5,E10.3))
  816 FORMAT(5E10.3,6I5)
  900 FORMAT(1H1)
  904 FORMAT(5X,'INPUT DATA:'/5X,'CALIBRATION CONTROL CODES:'/
     *        5X,'----------------------------'//
     *        15X,'NUMBER OF DIMENSIONS                   =',I4/
     *        15X,'MAXIMUM NUMBER OF FUNCTION EVALUATIONS  =',I4/
     *        15X,'OUTPUT DATA PRINT CODE                 =',I4/
     *        15X,'CONVERGENCE CRITERION CODE             =',I4/
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118

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     *         15X,'CUBIC INTERPOLATION OPTION CODE         =',I4/
     *         15X,'HESSIAN CONDITIONING OPTION CODE         =',I4/
     *         15X,'# OF NON-ZERO HESSIAN VALUES TO BE READ  =',I4/
     *         15X,'PARAMETER SCALING OPTION CODE            =',I4/
     *         15X,'# OF IMPROVED STARTING ESTIMATES REQUIRED =',I4/
     *         15X,'ABSOLUTE CONVERGENCE CRITERION FOR F(X)  =',1PE10.3/
     *         15X,'ABSOLUTE CONVERGENCE CRITERION FOR S(X)  =', E10.3/
     *         15X,'VALUE OF THE LINE-SEARCH PARAMETER, SIGMA =', E10.3/
     *         15X,'VALUE OF THE BRACKET-CHECK PARAMETER, ROA =', E10.3/
     *         15X,'VALUE OF THE BRACKET-CHECK PARAMETER, TAU =', E10.3/
     *         15X,'VALUE OF THE LINE SEARCH EXIT CRITERION  =', E10.3/
     *         15X,'VALUE OF THE FAIL-SAFE GLOBAL CRITERION  =', E10.3/
     *         15X,'# OF FN-EVALUATIONS BEFORE FAIL-SAFE EXIT =',I4/
     *         15X,'ERROR BOUND: MACHINE REPRESENTATION OF XX =', E10.3/
     *         15X,'ERROR BOUND: MACHINE REPRESENTATION OF FX =', E10.3/
     *         15X,'MINIMUM VALUE OF THE DIFFERENCE INTERVALS =', E10.3/
     *         15X,'MAXIMUM ERROR IN FWD-DIFFERENCE CALCULATN =', E10.3/
     *         15X,'SCALING FACTOR FOR THE OBJECTIVE FUNCTION =', E10.3/)
 908 FORMAT(/5X,'INPUT DATA:'/5X,'INITIAL VALUES OF THE FUNCTION ',
     *         'VARIABLES X1,X2,...,XN:'/5X,54('-')/10X,1P12E10.2)
 912 FORMAT(/5X,'INPUT DATA:'/5X,'LOWER AND UPPER BOUNDS ON THE ',
     *         'FUNCTION VARIABLES X1,X2,...,XN:'/5X,62('-'))
 916 FORMAT(10X,1P12E10.2)
 920 FORMAT(/5X,'INPUT DATA:'/5X,'INITIAL VALUES OF THE ',
     *         'DIFFERENCING INTERVALS D1,D2,...,DN:'/5X,58('-')
     *         /10X,1P12E10.2)
 924 FORMAT(/5X,'INPUT DATA:'/5X,'ABSOLUTE CONVERGENCE CRITERIA FOR ',
     *         'THE FUNCTION VARIABLES X1,X2,...,XN:'/5X,70('-')
     *         /10X,1P12E10.2)
 928 FORMAT(/5X,'INPUT DATA:'/5X,'SCALING FACTORS FOR THE ',
     *         'FUNCTION VARIABLES X1,X2,...,XN:'/5X,56('-')
     *         /10X,1P12E10.2)
 932 FORMAT(/5X,'INPUT DATA:'/5X,'INITIAL VALUE OF THE INVERSE ',
     *         'HESSIAN MATRIX:'/5X,44('-'))
 936 FORMAT(10X,1P12E10.2)
 940 FORMAT(1H1,4X,'OUTPUT DATA:'/5X,'THE RESULTS OF THE OPTIMIZATION ',
     *         'ROUTINE FOLLOW:'/5X,47('-')//5X,'#-I',1X,'#-F',1X,'#-G',
     *         9X,'F1',9X,'F2',9X,'S1',9X,'S2',9X,'X1',9X,'X2',9X,'X3',
     *         9X,'X4',9X,'X5',9X,'X6'/
     *         4X,3(1X,'---'),10(5X,'------')/)
 944 FORMAT(1H1,4X,'OUTPUT DATA:'/5X,'THE RESULTS OF THE OPTIMIZATION ',
     *         'ROUTINE FOLLOW:'/5X,47('-')//5X,'#-I',1X,'#-F',1X,'#-G',
     *         9X,'F1',9X,'F2',9X,'S1',9X,'S2',6X,'X1/X7',6X,'X2/X8',
     *         6X,'X3/X9',5X,'X4/X10',5X,'X5/X11',5X,'X6/X12'/
     *         4X,3(1X,'---'),10(5X,'------')/)
C
C     READ CALIBRATION SPECIFICATIONS
C
      READ(5,800) NDIM,NFMX,IPRN,ICON,
     *            ICUB,IHES,NHES,ISCA,NSLN
      READ(5,804) EPFA,EPSA, C1,C2,C3,EPXL,EPFL,NFLX
      READ(5,808) ETAF,ETAX,DMIN,FDER,SFUN
      READ(5,808) (X2(II),II=1,NDIM)
      READ(5,808) (XL(II),II=1,NDIM)
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119

```fortran
      READ(5,808) (XU(II),II=1,NDIM)
      READ(5,808) (DX(II),II=1,NDIM)
      IF(ISCA.EQ.1) READ(5,808) (SF(II),II=1,NDIM)
      IF(ICON.EQ.1) READ(5,808) (XE(II),II=1,NDIM)
C
C     INITIALIZE INVERSE HESSIAN
C
      DO 100 II=1,NDIM
      DO 100 JJ=1,NDIM
      H2(II,JJ)=0.0
  100 IF(II.EQ.JJ) H2(II,JJ)=-1.0
      IF(NHES.EQ.0) GO TO 130
      A1=FLOAT(NHES)/4.0
      A2=AINT(A1)
      IF(A1-A2.GT.0.0) A2=A2+1.0
      NH=IFIX(A2)
      NX=NHES-4*(NH-1)
      DO 110 II=1,NH
      IF(II.LT.NH) READ(5,812) ((IR,IC,H2(IR,IC)),JJ=1, 4)
  110 IF(II.EQ.NH) READ(5,812) ((IR,IC,H2(IR,IC)),JJ=1,NX)
      DO 120 II=1,NDIM
      DO 120 JJ=1,NDIM
      IF(H2(II,JJ).NE.0.0) H2(II,JJ)=-H2(II,JJ)
  120 IF(H2(II,JJ).NE.0.0) H2(JJ,II)= H2(II,JJ)
  130 CONTINUE
C
C     COMPUTE IMPROVED INITIAL ESTIMATES
C
      IF(NSLN.EQ.0) GO TO 200
      DO 190 II=1,NSLN
      READ(5,816) PLAM,PKAP,PCON,PFAL,RI,KR
      IF(RI.EQ.0.0) RI=2.50
      CALL NEWTON(PLAM,PKAP,PCON,PFAL,RI)
      GO TO (140,160), KR
  140 DO 150 JJ=1,NDIM
  150 IF(KOPT(JJ).EQ. 9) X2(JJ)=RI
      GO TO 180
  160 RC=0.0
      DO 170 JJ=1,NDIM
      IF(KOPT(JJ).EQ.12) KK=JJ
  170 IF(KOPT(JJ).EQ. 9) RC=X2(JJ)
      IF(RC.EQ.0.0) RC=PRP1(9)
      X2(KK)=RI/RC
  180 CONTINUE
  190 CONTINUE
  200 CONTINUE
C
C     ASSIGN DEFAULT VALUES
C
      IF(C1.EQ.0.0) C1=9.00E-01
      IF(C2.EQ.0.0) C2=1.00E-04
      IF(C3.EQ.0.0) C3=1.00E-01
      IF(EPXL.EQ.0.0) EPXL=1.00E-04
      IF(ETAF.EQ.0.0) ETAF=1.00E-04
```

```
        IF(ETAX.EQ.0.0) ETAX=1.00E-08
        IF(DMIN.EQ.0.0) DMIN=5.00E-03
        IF(FDER.EQ.0.0) FDER=1.00E-04
        IF(SFUN.EQ.0.0) SFUN=1.00E+00
        SMAL=1.00E-06
        DO 210 II=1,NDIM
        XT=X2(II)+DX(II)
        IF(XT.LE.XL(II)) DX(II)=XL(II)-X2(II)+SMAL
  210 IF(XT.GE.XU(II)) DX(II)=XU(II)-X2(II)-SMAL
C
C     PRINT INPUT DATA
C
        WRITE(6,900)
        WRITE(6,904) NDIM,NFMX,IPRN,ICON,ICUB,IHES,NHES,
       *             ISCA,NSLN,EPFA,EPSA, C1,C2,C3,EPXL,
       *             EPFL,NFLX,ETAX,ETAF,DMIN,FDER,SFUN
        WRITE(6,908) (X2(II),II=1,NDIM)
        WRITE(6,912)
        WRITE(6,916) (XL(II),II=1,NDIM)
        WRITE(6,916) (XU(II),II=1,NDIM)
        WRITE(6,920) (DX(II),II=1,NDIM)
        IF(ICON.EQ.1) WRITE(6,924) (XE(II),II=1,NDIM)
        IF(ISCA.NE.0) WRITE(6,928) (SF(II),II=1,NDIM)
        IF(NHES.EQ.0) GO TO 230
        WRITE(6,932)
        DO 220 II=1,NDIM
  220 WRITE(6,936) (-H2(II,JJ),JJ=1,NDIM)
  230 IF(NDIM.LE.6) WRITE(6,940)
        IF(NDIM.GT.6) WRITE(6,944)
        RETURN
        END
C
C
C
        SUBROUTINE NEWTON(PRP1,PRP2,PCON,PFAL,RI)
C
C     THIS SUBROUTINE COMPUTES AN INITIAL ESTIMATE
C     OF THE SURFACE SHAPE PARAMETER R BY MEANS OF THE
C     MODIFIED-NEWTON ROOT FINDING SCHEME.
C
C     WRITTEN BY J.S. DE NATALE,
C     DEPARTMENT OF CIVIL ENGINEERING,
C     UNIVERSITY OF CALIFORNIA, DAVIS.
C     VERSION I: MAY 1982.
C
        NI=0
        RO=RI
        DR=0.0
        ER=1.0E-06
        ALPH=PFAL/PCON
        BETA=PRP1/(PRP2-PRP1)
        A1=ALPH**BETA
        A2=ALPH**(2.0*BETA)
  100 R1=RI
```

```
      NI=NI+1
      IF(NI.GT.100) RI=RO
      IF(NI.GT.100) RETURN
  110 FF=2.0*(A1-A2)+R1*(A2-2.0)+2.0*R1**2-R1**3
      GG=A2-2.0+4.0*R1-3.0*R1**2
      HH=4.0-6.0*R1
      UF=FF/GG
      UG=1.0-FF*HH/GG**2
      DR=UF/UG
      R1=R1-DR
      IF(ABS(DR).LE.ER) GO TO 120
      GO TO 110
C
C     IF THE COMPUTED VALUE OF R IS
C     BETWEEN 1.50 AND 3.50, THEN STOP
C
  120 IF(R1.GE.1.5) GO TO 130
      RI=RI+1.0
      GO TO 100
  130 IF(R1.LE.3.5) GO TO 140
      RI=RI-0.5
      GO TO 100
  140 RI=R1
      RETURN
      END
C
C
C

      SUBROUTINE SEARCH
C
C     THIS SUBROUTINE DIRECTS THE LINE-SEARCH
C     COMPONENT OF THE GLOBAL MINIMIZATION ALGORITHM.
C     THE LOGIC IS PATTERNED AFTER THAT OF FLETCHER (1980).
C
C     WRITTEN BY J.S. DE NATALE,
C     DEPARTMENT OF CIVIL ENGINEERING,
C     UNIVERSITY OF CALIFORNIA, DAVIS.
C     VERSION I: MAY 1982.
C
      COMMON/BLK2/NDIM,NFMX,C1,C2,C3
      COMMON/BLK3/NI,NF,NG,F1,F2,S1,S2,X1(17),X2(17)
      COMMON/BLK4/EPFA,EPSA,ICON,JCON,IPRN,ICUB,XE(17)
      COMMON/BLK5/D1(17),D2(17),G1(17),G2(17),DG(17),DX(17),
     *            H1(17,17),H2(17,17)
      COMMON/BLK6/EPFL,EPXL,NFLX
      COMMON/BLK7/SFUN,SF(17),XL(17),XU(17)
      DIMENSION   DM(17),XM(17)
C
C     INITIALIZE AND UPDATE PARAMETERS
C
      FEND=F2
      NI=NI+1
      A1=0.0
      A2=1.00E+06
```

122

```
      F1=F2
      S1=S2
      DO 100 II=1,NDIM
      D1(II)=D2(II)
      G1(II)=G2(II)
      X1(II)=X2(II)
      DO 100 JJ=1,NDIM
  100 H1(II,JJ)=H2(II,JJ)
C
C     ESTABLISH AN INITIAL VALUE OF ALPHA
C
      IF(NI.EQ. 1) DF=F1
      Z1=10.0*EPFA
      IF(DF.LT.Z1) DF=Z1
      AA= 1.0
      Z1=-2.0*DF/S1
      IF(AA.GT.Z1) AA=Z1
      IF(LEND.EQ.1.AND.LPAS.EQ.0) AA=1.0
C
C     EVALUATE THE FUNCTION F(X1,X2,...,XN),
C     BUT TERMINATE THE LINE SEARCH IF THE CHANGE IN
C     X1,X2,...,XN IS SMALL
C
      LPAS=0
  200 IF(NF.GE.NFMX) GO TO 500
      LEND=1
      DO 210 II=1,NDIM
      X3=XL(II)+(XU(II)-XL(II))*(SIN(X2(II)))**2
      DX(II)=AA*D1(II)
      X2(II)=X1(II)+DX(II)
      X4=XL(II)+(XU(II)-XL(II))*(SIN(X2(II)))**2
      XC=ABS((X3-X4)/X3)
  210 IF(XC.GT.EPXL) LEND=0
      IF(LEND.EQ.1.AND.LPAS.EQ.1) GO TO 420
      F2=FFUN(X2)
      NF=NF+1
C
C     CHECK THE FAIL-SAFE EXIT CRITERION
C     AND TERMINATE THE GLOBAL SEARCH IF THE
C     FUNCTION IS NOT BEING SIGNIFICANTLY REDUCED
C
      IF((F1-F2).GT.EPFL) IX=NF
      IF((NF-IX).GT.NFLX) GO TO 500
      IF(LEND.EQ.1) GO TO 300
C
C     CHECK TO SEE THAT ALPHA IS WITHIN THE
C     ACCEPTABLE INTERVAL.  CHECK UPPER BOUND
C
      DF=F1-F2
      Z1=  -C2*AA*S1
      IF(DF.GE.Z1) GO TO 300
C
C     CHECK FAILS.  COMPUTE AN ACCEPTABLE VALUE BY
C     QUADRATIC INTERPOLATION
```

```
C
      IF(IPRN.EQ.3) CALL PRNOUT(1,NDIM)
      Z1=AA-A1
      Z2=2.0*(1.0+DF/(Z1*S1))
      AH=A1+Z1/Z2
C
C     UPDATE BRACKET
C     ENSURE THAT ALPHA IS NOT NEAR THE EXTREMES
C
      A2=AA
      Z2=C3*Z1
      IF(AH.LT.(A1+Z2)) AH=A1+Z2
      IF(AH.GT.(A2-Z2)) AH=A2-Z2
      AA=AH
      GO TO 200
C
C     CHECK SUCCEEDS.  EVALUATE THE GRADIENT AND SLOPE
C
  300 DO 310 II=1,NDIM
      IG=II
      DM(II)=DX(II)
      XM(II)=X2(II)
  310 G2(IG)=GFUN(IG,0,NDIM)
      NG=NG+1
      S2=DTPROD(G2,D1,NDIM)
      FM=F2
      SM=S2
      IF(LEND.EQ.1) GO TO 400
C
C     CHECK TO SEE THAT ALPHA IS WITHIN THE
C     ACCEPTABLE INTERVAL.  CHECK LOWER BOUND
C
      LPAS=1
      IF(S2.GE.(C1*S1)) GO TO 350
C
C     CHECK FAILS.  COMPUTE AN ACCEPTABLE VALUE
C     BY QUADRATIC EXTRAPOLATION
C
      IF(IPRN.EQ.3) CALL PRNOUT(1,NDIM)
      Z1=AA-A1
      Z2=S2/(S1-S2)
      AH=AA+Z1*Z2
C
C     UPDATE BRACKET
C     ENSURE THAT ALPHA IS NOT NEAR THE EXTREMES
C
      Z2=C3*Z1
      IF(AH.LT.(AA+Z2)) AH=AA+Z2
      Z2=9.0*Z1
      IF(AH.GT.(AA+Z2)) AH=AA+Z2
      Z2=(A2-AA)/2.0
      IF(AH.GT.(AA+Z2)) AH=AA+Z2
      A1=AA
      AA=AH
```

124

```
      F1=F2
      S1=S2
      GO TO 200
C
C     RE-CHECK LOWER BOUND IF THE
C     STRICT TERMINATION CRITERION IS EMPLOYED
C
  350 IF(ICUB.EQ.1) GO TO 400
      IF(ABS(S2).LE.(-C1*S1)) GO TO 400
C
C     CHECK FAILS.  COMPUTE AN ACCEPTABLE VALUE
C     BY CUBIC INTERPOLATION
C
      IF(IPRN.EQ.3) CALL PRNOUT(1,NDIM)
      Z1=AA-A1
      DD=(2.0*(F1-F2)+Z1*(S1+S2))/(Z1**3)
      CC=(S2-S1+3.0*DD*(A1**2-AA**2))/(2.0*Z1)
      BB=(AA*S1-A1*S2+3.0*DD*A1*AA*Z1)/Z1
      AH=BB/(-CC-SQRT(CC*CC-3.0*BB*DD))
C
C     UPDATE BRACKET
C     ENSURE THAT ALPHA IS NOT NEAR THE EXTREMES
C
      A2=AA
      Z2=C3*Z1
      IF(AH.LT.(A1+Z2)) AH=A1+Z2
      IF(AH.GT.(A2-Z2)) AH=A2-Z2
      AA=AH
      GO TO 200
C
C     TERMINATE THE LINE SEARCH.  CHECK FOR CONVERGENCE
C
  400 CONTINUE
      IF(IPRN.GE.2) CALL PRNOUT(1,NDIM)
      IF(LEND.EQ.0) GO TO 440
      IF(LPAS.EQ.1.OR.F2.LE.F1) GO TO 420
      F2=F1
      S2=S1
      DO 410 II=1,NDIM
      R7=X1(II)
      X1(II)=X2(II)
      X2(II)=R7
      R7=G1(II)
      G1(II)=G2(II)
  410 G2(II)=R7
      GO TO 440
  420 F2=FM
      S2=SM
      DO 430 II=1,NDIM
      DX(II)=DM(II)
  430 X2(II)=XM(II)
  440 CONTINUE
      DF=FEND-F2
      JCON=0
```

```
              CALL CONCHK(DF,S2,DX,NDIM)
              IF(IPRN.EQ.1.AND.JCON.EQ.1)
           *                  CALL PRNOUT(1,NDIM)
              IF(JCON.EQ.1) CALL PRNOUT(5,NDIM)
              RETURN
          500 CONTINUE
              JCON=1
              CALL PRNOUT(1,NDIM)
              IF((NF-IX).LE.NFLX) CALL PRNOUT(2,NDIM)
              IF((NF-IX).GT.NFLX) CALL PRNOUT(4,NDIM)
              RETURN
              END
      C
      C
      C
              SUBROUTINE BFGSUP(S2,IH,NI,NDIM)
      C
      C       THIS SUBROUTINE UPDATES THE HESSIAN MATRIX BY
      C       MEANS OF THE BFGS (BROYDEN-FLETCHER-GOLDFARB-SHANNO)
      C       UPDATING FORMULA.  THE NEW LINE-SEARCH DIRECTION
      C       IS ALSO COMPUTED.
      C
      C       WRITTEN BY J.S. DE NATALE,
      C       DEPARTMENT OF CIVIL ENGINEERING,
      C       UNIVERSITY OF CALIFORNIA, DAVIS.
      C       VERSION I: MAY 1982.
      C
              COMMON/BLK5/D1(17),D2(17),G1(17),G2(17),DG(17),DX(17),
           *              H1(17,17),H2(17,17)
              DIMENSION   SV(17),SM(17,17)
      C
      C       UPDATE THE INVERSE HESSIAN MATRIX
      C
              DO 100 II=1,NDIM
          100 DG(II)=G2(II)-G1(II)
              CALL MVPROD(H1,DG,SV,NDIM)
              ALPH=DTPROD(DG,SV,NDIM)
              AS=1.0
              IF(ALPH.GT.0.0) AS=-AS
              BETA=DTPROD(DX,DG,NDIM)
              IF(IH.EQ.0) GO TO 120
              IF(NI.EQ.1) IC=0
              IF(NI.GT.1.AND.IC.EQ.0) GO TO 120
              GAMA= -BETA/ALPH
              DO 110 II=1,NDIM
              DO 110 JJ=1,NDIM
          110 H1(II,JJ)=H1(II,JJ)*GAMA
          120 CONTINUE
              DO 130 II=1,NDIM
          130 SV(II)=SQRT(ABS(ALPH))*(DX(II)/BETA-SV(II)/ALPH)
              CALL VVPROD(SV,SV,SM,NDIM)
              DO 140 II=1,NDIM
              DO 140 JJ=1,NDIM
          140 H2(II,JJ)=H1(II,JJ)-SM(II,JJ)*AS
```

126

```
      CALL MVPROD(H1,DG,SV,NDIM)
      CALL VVPROD(SV,DG,SM,NDIM)
      CALL MMPROD(SM,H1,SV,NDIM)
      DO 150 II=1,NDIM
      DO 150 JJ=1,NDIM
  150 H2(II,JJ)=H2(II,JJ)-H1(II,JJ)/ALPH
      CALL VVPROD(DX,DX,SM,NDIM)
      DO 160 II=1,NDIM
      DO 160 JJ=1,NDIM
  160 H2(II,JJ)=H2(II,JJ)-SM(II,JJ)/BETA
C
C     COMPUTE A NEW SEARCH DIRECTION
C
      CALL MVPROD(H2,G2,D2,NDIM)
C
C     IF THE NEW COMPUTED SEARCH DIRECTION IS NOT
C     A DESCENT DIRECTION, SET THE INVERSE HESSIAN
C     TO THE IDENTITY MATRIX AND RESTART THE SEARCH
C
      IC=0
      S2=DTPROD(G2,D2,NDIM)
      IF(S2.LT.0.0) GO TO 190
      IC=1
      DO 170 II=1,NDIM
      DO 170 JJ=1,NDIM
  170 H2(II,JJ)= 0.0
      DO 180 II=1,NDIM
      IG=II
      G2(IG)=GFUN(IG,1,NDIM)
  180 H2(II,II)=-1.0
      CALL MVPROD(H2,G2,D2,NDIM)
      CALL PRNOUT(3,NDIM)
      S2=DTPROD(G2,D2,NDIM)
  190 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE CONCHK(FV,SV,XV,ND)
C
C     THIS SUBROUTINE CHECKS FOR CONVERGENCE.
C
      COMMON/BLK4/EPFA,EPSA,ICON,JCON,IPRN,ICUB,XE(17)
      DIMENSION   XV(17)
      JCON=1
      GO TO (100,200,300), ICON
  100 DO 110 II=1,ND
      IF(ABS(XV(II)).GT.XE(II)) JCON=0
      IF(JCON.EQ.0) RETURN
  110 CONTINUE
      RETURN
  200 IF(ABS(FV).GT.EPFA) JCON=0
      RETURN
```

```
  300 IF(ABS(SV).GT.EPSA) JCON=0
      RETURN
      END
C
C
C

      SUBROUTINE PRNOUT(IP,ND)
C
C     THIS SUBROUTINE CONTROLS THE
C     PRINTING OF THE MINIMIZATION SEARCH RESULTS.
C
      COMMON/BLK3/NI,NF,NG,F1,F2,S1,S2,X1(17),X2(17)
      COMMON/BLK7/SFUN,SF(17),XL(17),XU(17)
      DIMENSION   XS(17)
      GO TO (100,200,300,400,500), IP
  100 DO 110 II=1,ND
      XS(II)=X2(II)/SF(II)
  110 XS(II)=XL(II)+(XU(II)-XL(II))*(SIN(XS(II)))**2
      IF(ND.LE.6) WRITE(6,120) NI,NF,NG,F1,F2,S1,S2,
     *                         (XS(II),II=1,ND)
      IF(ND.GT.6) WRITE(6,130) NI,NF,NG,F1,F2,S1,S2,
     *                         (XS(II),II=1,ND)
  120 FORMAT(5X,I3,2I4,1P10E11.3)
  130 FORMAT(5X,I3,2I4,1P10E11.3/60X,6E11.3)
      RETURN
  200 WRITE(6,210)
  210 FORMAT(/5X,'***** NO ADDITIONAL ',
     *       'FUNCTION EVALUATIONS ARE PERMITTED *****'/)
      RETURN
  300 WRITE(6,310)
  310 FORMAT(/5X,'***** THE INVERSE HESSIAN HAS ',
     *       'BEEN RESET TO THE IDENTITY MATRIX *****'/)
      RETURN
  400 WRITE(6,410)
  410 FORMAT(/5X,'***** THE SEARCH HAS BEEN TERMINATED ',
     *       'DUE TO INSUFFICIENT PROGRESS *****'/)
      RETURN
  500 WRITE(6,510)
  510 FORMAT(/5X,'***** THE SEARCH HAS BEEN ',
     *       'TERMINATED AS A RESULT OF CONVERGENCE *****'/)
      RETURN
      END
C
C
C
      FUNCTION DTPROD(V1,V2,ND)
      DIMENSION   V1(17),V2(17)
      DTPROD=0.0
      DO 100 II=1,ND
  100 DTPROD=DTPROD+V1(II)*V2(II)
      RETURN
      END
C
C
```

```
C
      SUBROUTINE VVPROD(V1,V2,V3,ND)
      DIMENSION  V1(17),V2(17),V3(17,17)
      DO 100 II=1,ND
      DO 100 JJ=1,ND
  100 V3(II,JJ)=V1(II)*V2(JJ)
      RETURN
      END
C
C
C
      SUBROUTINE MVPROD(V3,V1,V2,ND)
      DIMENSION  V1(17),V2(17),V3(17,17)
      DO 100 II=1,ND
      V2(II)=0.0
      DO 100 JJ=1,ND
  100 V2(II)=V2(II)+V3(II,JJ)*V1(JJ)
      RETURN
      END
C
C
C
      SUBROUTINE MMPROD(V3,V4,V1,ND)
      DIMENSION  V1(17),V3(17,17),V4(17,17)
      DO 110 JJ=1,ND
      DO 100 II=1,ND
      V1(II)=0.0
      DO 100 KK=1,ND
  100 V1(II)=V1(II)+V3(II,KK)*V4(KK,JJ)
      DO 110 II=1,ND
  110 V4(II,JJ)=V1(II)
      RETURN
      END
C
C
C
      FUNCTION FFUN(XX)
C
C     THIS FUNCTION SUBROUTINE ESTABLISHES
C     THE VALUE OF THE OBJECTIVE FUNCTION AT
C     THE SPECIFIED LOCATION.
C
      COMMON/BLK7/SFUN,SF(17),XL(17),XU(17)
      DIMENSION   XX(17)
      CALL EVAL(XX,FF,0)
      FFUN=FF*SFUN
      RETURN
      END
C
C
C
      FUNCTION GFUN(IV,IR,ND)
C
C     THIS FUNCTION SUBROUTINE ESTIMATES THE GRADIENT OF
```

```
C       THE OBJECTIVE FUNCTION BY MEANS OF FINITE DIFFERENCES.
C       THE LOGIC FOLLOWS THAT OUTLINED BY STEWART (1967).
C
C       WRITTEN BY J.S. DE NATALE,
C       DEPARTMENT OF CIVIL ENGINEERING,
C       UNIVERSITY OF CALIFORNIA, DAVIS.
C       VERSION I: MAY 1982.
C
        COMMON/BLK1/ETAF,ETAX,DMIN,FDER
        COMMON/BLK3/NI,NF,NG,F1,F2,S1,S2,X1(17),X2(17)
        COMMON/BLK5/D1(17),D2(17),G1(17),G2(17),DG(17),DX(17),
      *             H1(17,17),H2(17,17)
        DIMENSION    XX(17)
C
C       COMPUTE INITIAL GRADIENT VECTOR.  RECOMPUTE
C       INITIAL GRADIENT VECTOR IF THE SEARCH HAS BEEN RESTARTED
C
        DO 100 II=1,ND
  100 XX(II)=X2(II)
        IF(IR.EQ.0) GO TO 110
        DD=DMIN
        GO TO 160
  110 IF(NG.GT.0) GO TO 120
        XX(IV)=X2(IV)+DX(IV)
        Z1=FFUN(XX)
        GFUN=(Z1-F2)/DX(IV)
        NF=NF+1
        RETURN
C
C       COMPUTE SUBSEQUENT GRADIENT VECTORS
C
  120 CONTINUE
        R1=1.0E-10
        R2=1.0E+00
        IF(ABS(G1(IV)).LT.R1) G1(IV)=R1*SIGN(R2,G1(IV))
C
C       ESTABLISH GOVERNING ERROR BOUND
C
        ETAA=ETAF
        Z1=ETAX*ABS(G1(IV)*X2(IV)/F2)
        IF(ETAA.LT.Z1) ETAA=Z1
C
C       ESTABLISH DIFFERENCING INTERVAL
C
        Z1=G1(IV)**2
        Z2=ETAA*ABS(H2(IV,IV)*F2)
        IF(Z1.LT.Z2) GO TO 130
        Z1=ABS(F2/H2(IV,IV))**0.5
        DO=2.0*Z1
        Z1=DO*ABS(H2(IV,IV))
        Z2=3.0*Z1+4.0*ABS(G1(IV))
        DD=DO*(1.0-Z1/Z2)
      ` GO TO 140
  130 Z1=ABS(F2*G1(IV)/H2(IV,IV)**2)**(1.0/3.0)
```

130

```
              DO=2.0*Z1
              Z1=2.0*ABS(G1(IV))
              Z2=2.0*ABS(H2(IV,IV))*DO+2.0*Z1
              DD=DO*(1.0-Z1/Z2)
      140 IF(ABS(DD).LT.DMIN) DD=DMIN*SIGN(R2,DD)
              DD=DD*SIGN(R2,G1(IV))*SIGN(R2,-H2(IV,IV))
C
C       DECIDE ON THE MOST
C       APPROPRIATE DIFFERENCING FORMULA TO EMPLOY
C
              Z1=ABS(H2(IV,IV)*DD/G1(IV))/2.0
              IF(Z1.GE.FDER) GO TO 150
C
C       FORWARD DIFFERENCES
C
              XX(IV)=X2(IV)+DD
              Z1=FFUN(XX)
              GFUN=(Z1-F2)/DD
              NF=NF+1
              RETURN
C
C       CENTRAL DIFFERENCES
C
      150 AA=ABS(H2(IV,IV))/2.0
              BB=ABS(G1(IV))
              CC=-ETAA*ABS(F2)/FDER
              DD=(2.0*CC)/(-BB-SQRT(BB*BB-4.0*AA*CC))
              IF(ABS(DD).LT.DMIN) DD=DMIN*SIGN(R2,DD)
      160 XX(IV)=X2(IV)+DD
              Z1=FFUN(XX)
              NF=NF+1
              XX(IV)=X2(IV)-DD
              Z2=FFUN(XX)
              NF=NF+1
              GFUN=(Z1-Z2)/(2.0*DD)
              RETURN
              END
C
C
C
              SUBROUTINE OPEN
C
C       THIS SUBROUTINE ASSIGNS AND OPENS FILES 5
C       AND 6.  PROMPTS ARE SENT TO THE TERMINAL (FILE 4).
C
              DIMENSION  IFIL(21)
              WRITE(4,100)
      100 FORMAT(1X,'INPUT DATA FILENAME=?',$)
              READ(4,110) IFIL
      110 FORMAT(21A2)
              OPEN(UNIT=5,NAME=IFIL,TYPE='OLD')
              WRITE(4,120)
      120 FORMAT(1X,'OUTPUT DATA PRINT FILENAME=?',$)
              READ(4,110) IFIL
```

131

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      OPEN(UNIT=6,NAME=IFIL,TYPE='NEW')
      RETURN
      END
C
C
C
      SUBROUTINE EXIT
C
C     THIS SUBROUTINE CLOSES FILES 5 AND 6
C     AND TERMINATES EXECUTION OF THE PROGRAM.
C
      CLOSE(UNIT=5)
      CLOSE(UNIT=6)
      STOP
      END
C
C
C
C     ****************************************************
C     *                                                  *
C     *    THE REMAINING TWO SUBROUTINES -- CLAY &        *
C     *    BOUNDS -- ARE A NUMERICAL IMPLEMENTATION       *
C     *    OF THE GOVERNING CONSTITUTIVE EQUATIONS        *
C     *    FOR THE BOUNDING SURFACE SOIL PLASTICITY       *
C     *    MODEL                                          *
C     *                                                  *
C     ****************************************************
C
      SUBROUTINE CLAY(IDIM,INC,ITNO,PROP,STOR,SIGBM,EPM,
     *                DSIGM,DEPM,C,UB,DLTAU,GAM,KIND,LARGE)
C
C     THIS SUBROUTINE EVALUATES YANNIS DAFALIAS'
C     BOUNDING SURFACE PLASTICITY MODEL FOR CLAY SOILS.
C
C     WRITTEN BY L.R. HERRMANN,
C     DEPARTMENT OF CIVIL ENGINEERING,
C     UNIVERSITY OF CALIFORNIA, DAVIS.
C     LAST REVISED: JULY 1982.
C
      DIMENSION PROP(21),STOR(6),SIGB(6),DSIG(6),DEP(6),C(6,6),
     *          SB(3,3),SF(3,3),II(6),DLTA(3,3),DEPM(6),
     *          SIGBM(6),DSIGM(6),DEPT(3,3),EPM(6),EPB(6)
      DATA II/11,22,33,12,13,23/, DLTA/1.0,3*0.0,1.0,3*0.0,1.0/
      ALFUN(CV,RT,SINV)=2.0*RT*CV/(1.0+RT-(1.0-RT)*SINV)
      SMALL=0.0001*PROP(8)
      DO 100 I=1,6
      SIGB(I)=SIGBM(I)
      DSIG(I)=DSIGM(I)
      EPB(I)=EPM(I)
  100 DEP(I)=DEPM(I)
      IF(ITNO.GT.1) GO TO 120
      IF(INC .GT.1) GO TO 110
C
C     INITIALIZE HISTORY
```

```
C
      STOR(1)=PROP(21)
      STOR(2)=STOR(1)
      STOR(3)=0.5*(SIGB(1)+SIGB(2))
      STOR(4)=0.01*PROP(8)
      STOR(5)=0.0
      GO TO 120
C
C     UPDATE HISTORY
C
  110 STOR(1)=STOR(2)
      STOR(3)=STOR(3)+STOR(4)
      STOR(5)=STOR(5)+STOR(6)
C
C     CONVERT FROM PLANE STRAIN TO 3-D
C
  120 IF(IDIM.EQ.3) GO TO 140
      SIGB(4)=SIGB(3)
      SIGB(3)=STOR(3)
      DSIG(4)=DSIG(3)
      DSIG(3)=STOR(4)
      DEP(4)=DEP(3)
      DEP(3)=0.0
      EPB(4)=EPB(3)
      EPB(3)=0.0
      DO 130 I=5,6
      SIGB(I)=0.0
      DSIG(I)=0.0
      EPB(I)=0.0
  130 DEP(I)=0.0
C
C     DETERMINE 3-D INCREMENTAL PROPERTIES
C
  140 GAM=PROP(6)
C
C     CALCULATE EFFECTIVE STRESS INVARIANTS AND
C     DISTORTIONAL STRESS AND CHANGE MATRIX COMPONENTS
C     TO TENSOR COMPONENTS.
C
      XIB=0.0
      XIF=0.0
      DDIL=0.0
      DILB=0.0
      DO 150 I=1,3
      DDIL=DDIL+DEP(I)
      DILB=DILB+EPB(I)
      XIB=XIB+SIGB(I)
  150 XIF=XIF+SIGB(I)+DSIG(I)
      VOIDB=1.0+PROP(20)
      VOIDF=VOIDB
      IF(LARGE.EQ.0) GO TO 160
      VOIDB=VOIDB*EXP(-DILB)
      VOIDF=VOIDF*EXP(-DILB-DDIL)
  160 DO 170 N=1,6
```

133

```
          I=II(N)/10
          J=MOD(II(N),10)
          SB(I,J)=SIGB(N)-XIB*DLTA(I,J)/3.0
          SB(J,I)=SB(I,J)
          DEPT(I,J)=DEP(N)*(1.0+DLTA(I,J))*0.5
          DEPT(J,I)=DEPT(I,J)
          SF(I,J)=SIGB(N)+DSIG(N)-DLTA(I,J)*XIF/3.0
      170 SF(J,I)=SF(I,J)
          GAMP=0.0
          IF(KIND .EQ. 0) GO TO 180
          GAMP=GAM
          UB=STOR(5)
          DLTAU=GAM*DDIL
      180 XIB=XIB-UB*3.0
          XIF=XIF-(UB+DLTAU)*3.0
          STOR(6)=DLTAU
          SRTJB=0.0
          SRTJF=0.0
          DO 190 I=1,3
          DO 190 J=1,3
          SRTJB=SRTJB+SB(I,J)*SB(I,J)
      190 SRTJF=SRTJF+SF(I,J)*SF(I,J)
          SRTJB=SQRT(0.5*SRTJB)
          SRTJF=SQRT(0.5*SRTJF)
          SCUB=0.0
          SCUF=0.0
          DO 200 I=1,3
          DO 200 J=1,3
          DO 200 K=1,3
          SCUB=SCUB+SB(I,J)*SB(J,K)*SB(K,I)
      200 SCUF=SCUF+SF(I,J)*SF(J,K)*SF(K,I)
          SCUB=SCUB/3.0
          SCUF=SCUF/3.0
          SN3AB=0.0
          IF(SRTJB.GT.SMALL) SN3AB=1.5*SQRT(3.0)*SCUB/SRTJB**3
          IF(SN3AB.GT. 1.0) SN3AB= 1.0
          IF(SN3AB.LT.-1.0) SN3AB=-1.0
          SN3AF=0.0
          IF(SRTJF.GT.SMALL) SN3AF=1.5*SQRT(3.0)*SCUF/SRTJF**3
          IF(SN3AF.GT. 1.0) SN3AF= 1.0
          IF(SN3AF.LT.-1.0) SN3AF=-1.0
          CS3AB=SQRT(1.0-SN3AB**2)
          CS3AF=SQRT(1.0-SN3AF**2)
      C
      C     AVOID ZERO MEAN PRESSURE
      C
          IF(ABS(XIB).GT.SMALL) GO TO 210
          DU=XIB
          XIB=SMALL
          IF(DU.LT.0.0) XIB=-SMALL
      210 IF(ABS(XIF).GT.SMALL) GO TO 220
          DU=XIF
          XIF=SMALL
          IF(DU.LT.0.0) XIF=-SMALL
```

134

```
  220 CONTINUE
C
C     CALCULATE ELASTIC PROPERTIES
C
      DU1=VOIDB/3.0/PROP(2)
      DU2=1.5*(1.0-2.0*PROP(5))/(1.0+PROP(5))
      DU=XIB
      IF(DU.LT.PROP(7)) DU=PROP(7)
      BB=DU1*DU
      GB=DU2*BB
      IF(PROP(5).GT.0.5) GB=PROP(5)
      DU1=VOIDF/3.0/PROP(2)
      DU=XIF
      IF(DU.LT.PROP(7)) DU=PROP(7)
      BF=DU1*DU
      GF=DU2*BF
      IF(PROP(5).GT.0.5) GF=GB
      DO 230 M=1,6
      I=II(M)/10
      J=MOD(II(M),10)
      DO 230 N=M,6
      K=II(N)/10
      L=MOD(II(N),10)
      DU1=DLTA(K,I)*DLTA(L,J)+DLTA(K,J)*DLTA(I,L)
      C(M,N)=(GB+GF)*DU1*0.5+0.5*(BB+BF+2.0*GAMP-2.0*(GB+GF)/3.0)
     *      *DLTA(I,J)*DLTA(K,L)
  230 C(N,M)=C(M,N)
C
C     CALCULATE SIZE OF BOUNDING SURFACE
C
      XIOB=STOR(1)
      XIOF=STOR(2)
      XIL=PROP(7)
      DU10=1.0/(PROP(1)-PROP(2))
      IF(XIOB.GE.XIL.AND.XIOF.GE.XIL) GO TO 240
      XIOBS=XIOB
      IF(XIOB.LT.XIL) XIOBS=XIL
      XIOFS=XIOF
      IF(XIOF.LT.XIL) XIOFS=XIL
      XIOF=XIOB+DU10*0.5*((XIOFS*VOIDF+XIOBS*VOIDB)*DDIL-
     *     (XIOBS*VOIDB/BB+XIOFS*VOIDF/BF)*(XIF-XIB)/3.0)
      GO TO 250
  240 XIOF=XIOB*EXP(DU10*0.5*((VOIDB+VOIDF)*DDIL-
     *     (VOIDB/BB+VOIDF/BF)*(XIF-XIB)/3.0))
  250 STOR(2)=XIOF
      IF(INC+ITNO.EQ.2) GO TO 410
C
C     CALCULATE BOUNDING SURFACE PROPERTIES
C
      CALL BOUNDS(PROP,SRTJB,SN3AB,XSB,XIOB,XIB,GAMB,DFIB,
     *            DFJB,XKSB,DFALB,DFJJB,BSB,VOIDB)
      CALL BOUNDS(PROP,SRTJF,SN3AF,XSF,XIOF,XIF,GAMF,DFIF,
     *            DFJF,XKSF,DFALF,DFJJF,BSF,VOIDF)
      DB=BSB-1.0
```

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      IF(DB .LT. 0.0) DB=0.0
      DF=BSF-1.0
      IF(DF.LT.0.0) DF=0.0
C
C     CALCULATE PLASTIC MODULUS
C     CHECK FOR ELASTIC ZONE AND UNLOADING
C
      XMS=ALFUN(PROP(17),PROP(19),SN3AB)
      DU7=0.0001**(1.0/XMS)
      LB=0
      DDD=1.0+DB*(1.0-PROP(15))
      IF(DDD.LE.0.0) GO TO 290
      LB=1
      H=ALFUN(PROP(16),PROP(18),SN3AB)
      DU=ABS(XSB)
      IF(DU.LT.DU7) DU=DU7
      DU8=9.0*DFIB**2+DFJB**2/3.0
      DU9=XIOB
      IF(XIOB.LT.XIL) DU9=XIL
      XKB=XKSB+H*DB/DDD*(1.0+1.0/DU**XMS)*DU8*DU9*DU10*VOIDB
      DU1=3.0*BB*DFIB
      DU2=GB*DFJJB
      DU2P=SQRT(3.0)*GB*DFALB
      DU3=XKB+9.0*BB*DFIB**2+GB*DFJB**2+GB*(DFALB*CS3AB)**2
      SUM=0.0
      T1=0.0
      IF(SRTJB.EQ.0.0) GO TO 280
      DO 270 I=1,3
      DO 270 J=1,3
      DU=0.0
      DO 260 K=1,3
  260 DU=DU+SB(I,K)*SB(K,J)
      T1=T1+(DU-1.5*SCUB*SB(I,J)/SRTJB**2)*DEPT(I,J)/SRTJB**2
  270 SUM=SUM+SB(I,J)*DEPT(I,J)
      T1=T1-2.0*DDIL/3.0
  280 DU=(DU1*DDIL+DU2*SUM+DU2P*T1)/DU3
      IF(DU.LT.0.0) LB=0
  290 LF=0
      DDD=1.0+DF*(1.0-PROP(15))
      IF(DDD.LE.0.0) GO TO 330
      LF=1
      H=ALFUN(PROP(16),PROP(18),SN3AF)
      DU=ABS(XSF)
      IF(DU.LT.DU7) DU=DU7
      DU8=9.0*DFIF**2+DFJF**2/3.0
      XMS=ALFUN(PROP(17),PROP(19),SN3AF)
      DU9=XIOF
      IF(XIOF.LT.XIL) DU9=XIL
      XKF=XKSF+H*DF/DDD*(1.0+1.0/DU**XMS)*DU8*DU9*DU10*VOIDF
      DU4=3.0*BF*DFIF
      DU5=GF*DFJJF
      DU6=XKF+9.0*BF*DFIF**2+GF*DFJF**2+GF*(DFALF*CS3AF)**2
      DU5P=GF*DFALF*SQRT(3.0)
      SUM=0.0
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136

```
      T1=0.0
      IF(SRTJF.EQ.0.0) GO TO 320
      DO 310 I=1,3
      DO 310 J=1,3
      DU=0.0
      DO 300 K=1,3
  300 DU=DU+SF(I,K)*SF(K,J)
      T1=T1+(DU-1.5*SCUF*SF(I,J)/SRTJF**2)*DEPT(I,J)/SRTJF**2
  310 SUM=SUM+SF(I,J)*DEPT(I,J)
      T1=T1-2.0*DDIL/3.0
  320 DU=(DU4*DDIL+DU5*SUM+DU5P*T1)/DU6
      IF(DU.LT.0.0) LF=0
C
C     CALCULATE PLASTIC PORTION OF INCREMENTAL PROPERTIES
C
  330 IF(LF+LB.EQ.0) GO TO 410
      DO 400 M=1,6
      I=II(M)/10
      J=MOD(II(M),10)
      DO 400 N=M,6
      K=II(N)/10
      L=MOD(II(N),10)
      DU=0.0
      IF(LB.EQ.0) GO TO 360
      T2=0.0
      T1=0.0
      IF((SRTJB**4).EQ.0.0) GO TO 350
      DO 340 LL=1,3
      T2=T2+SB(K,LL)*SB(LL,L)
  340 T1=T1+SB(I,LL)*SB(LL,J)
      T1=DU2P*(T1/SRTJB**2-1.5*SCUB*SB(I,J)/SRTJB**4-2.0*DLTA(I,J)/3.0)
      T2=DU2P*(T2/SRTJB**2-1.5*SCUB*SB(K,L)/SRTJB**4-2.0*DLTA(K,L)/3.0)
  350 DU=-0.5*(DU1*DLTA(I,J)+DU2*SB(I,J)+T1)*(DU1*DLTA(K,L)+
     *     DU2*SB(K,L)+T2)/DU3
      IF(LF.EQ.0) GO TO 390
  360 T2=0.0
      T1=0.0
      IF((SRTJF**4).EQ.0.0) GO TO 380
      DO 370 LL=1,3
      T2=T2+SF(K,LL)*SF(LL,L)
  370 T1=T1+SF(I,LL)*SF(LL,J)
      T1=DU5P*(T1/SRTJF**2-1.5*SCUF*SF(I,J)/SRTJF**4-2.0*DLTA(I,J)/3.0)
      T2=DU5P*(T2/SRTJF**2-1.5*SCUF*SF(K,L)/SRTJF**4-2.0*DLTA(K,L)/3.0)
  380 DU=DU-0.5*(DU4*DLTA(I,J)+DU5*SF(I,J)+T1)*(DU4*DLTA(K,L)+
     *      DU5*SF(K,L)+T2)/DU6
  390 C(M,N)=DU+C(M,N)
  400 C(N,M)=C(M,N)
  410 CONTINUE
      IF(IDIM.EQ.3) RETURN
C
C     CONVERT 3-D PROPERTIES TO PLANE STRAIN
C
      DU=0.0
      DO 420 I=1,4
```

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      DU=C(3,I)*DEP(I)+DU
      C(3,I)=C(4,I)
  420 C(4,I)=0.0
      DO 430 I=1,3
      C(I,3)=C(I,4)
  430 C(I,4)=0.0
      STOR(4)=DU
      RETURN
      END
C
C
C
      SUBROUTINE BOUNDS(PROP,SRTJ,SN3A,X,XIO,XI,GAM,
     *                  DFI,DFJ,XKS,DFAL,DFJJ,BS,VOID)
C
C     THIS SUBROUTINE EVALUATES THE RELATIONSHIP OF THE
C     STRESS STATE TO THE BOUNDING SURFACE, AS REQUIRED BY
C     SUBROUTINE CLAY.
C
C     WRITTEN BY L.R. HERRMANN,
C     DEPARTMENT OF CIVIL ENGINEERING,
C     UNIVERSITY OF CALIFORNIA, DAVIS.
C     LAST REVISED: JULY 1982.
C
      DIMENSION PROP(21),FSS(3)
      ALFUN(CV,RT,SINV)=2.0*RT*CV/(1.0+RT-(1.0-RT)*SINV)
      DFUN(FUN,RT,FUNC)=FUN**2*(1.0-RT)/(2.0*RT*FUNC)
      XN=ALFUN(PROP(3),PROP(4),SN3A)
      DNAL=DFUN(XN,PROP(4),PROP(3))
      R=ALFUN(PROP(9),PROP(12),SN3A)
      DRAL=DFUN(R,PROP(12),PROP(9))
      A=ALFUN(PROP(10),PROP(13),SN3A)
      DAAL=DFUN(A,PROP(13),PROP(10))
      YS=R*A/XN
      CC=PROP(14)
      IF(CC.GT.0.999) CC=0.999
C
C     SHIFT PROJECTION POINT
C
      D1=XI-XIO*CC
      IF(ABS(D1).LT.0.001) D1=0.001
      D2=CC-1.0/R
      D3=D1*D2
      D5=CC*(CC-2.0/R)
      Q =SRTJ/D1
      QC=1.0E+20
      IF(R*CC.NE.1.0) QC=XN/(1.0-R*CC)
      QO=1.0E+20
      IF(CC.NE.0.0) QO=XN*(SQRT(1.0+YS*YS)-(1.0+YS))/R/CC
      IF(SRTJ.NE.0.0) GO TO 100
      IF(D1.GT.  0.0) GO TO 120
      GO TO 140
  100 IF(CC.LT.1.0/R) GO TO 110
      IF(Q .GE.  0.0) GO TO 120
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138

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      IF(Q .LE.    QC) GO TO 120
      IF(Q .GE.    QO) GO TO 140
      GO TO 130
  110 IF(Q .GE.    QC) GO TO 130
      IF(Q .GE.    0.0) GO TO 120
      IF(Q .LE.    QO) GO TO 130
      GO TO 140
C
C     PROJECTION ON ELLIPSE 1
C
  120 D4=D1*D1+((R-1.0)*SRTJ/XN)**2
      BS=XIO*(-D3+SQRT(D3*D3-D4*(D5+(2.0-R)/R)))/D4
      LOST=1
      GO TO 150
C
C     PROJECTION ON HYPERBOLA
C
  130 D6=SRTJ*(1.0/R+A/XN)/XN
      D7=D3+D6
      D8=D1*D1-(SRTJ/XN)**2
      BS=-0.5*XIO*(D5-2.0*A/R/XN)/D7
      IF(D8.EQ.0.0) GO TO 150
      BS=XIO*(-D7+SQRT(D7*D7-D8*(D5-2.0*A/R/XN)))/D8
      LOST=2
      GO TO 150
C
C     PROJECTION ON ELLIPSE 2
C
  140 T=PROP(11)
      FOP=XN/SQRT(1.0+YS**2)
      XJO=A*(1.0+YS-SQRT(1.0+YS**2))/YS
      BT=T*(XJO-T*FOP)/(XJO-2.0*T*FOP)
      RO=(BT-T)/FOP/XJO
      PSI=1.0/(R*(BT-T))
      D9=T-BT+CC
      D10=D1*D9
      D11=D1*D1+RO*SRTJ*SRTJ
      BS=XIO*(-D10+SQRT(D10*D10-D11*(D9*D9-BT*BT)))/D11
      LOST=3
  150 XIBAR=BS*(XI-XIO*CC)+XIO*CC
      IF(XIBAR.EQ.0.0) XIBAR=1.0E-20
      TH=BS*SRTJ/XIBAR
      X=TH/XN
      DU=XIO
      IF(XIO.LT.PROP(7)) DU=PROP(7)
      DUS=12.0*VOID/(PROP(1)-PROP(2))*XIO**2*DU
      GO TO (200,300,400), LOST
C
C     NORMAL CONSOLIDATION ZONE
C
  200 PSI=YS/(R-1.0)**2
      DU=R*(1.0+X*X+R*(R-2.0)*X*X)
      GAM=(1.0+(R-1.0)*SQRT(1.0+R*(R-2.0)*X*X))/DU
      DFI=2.0*XIO*(GAM-1.0/R)*PSI
```

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      DFJJ=2.0*XIO*GAM*((R-1.0)/XN)**2*PSI*BS/XIBAR
      DFJ=DFJJ*SRTJ
      XKS=DUS*(GAM-1.0/R)*(GAM+R-2.0)*PSI*PSI/R
      DFAL=PSI*6.0*(R-1.0)*TH*GAM*XIO*(((R-1.0)/(R**2*
     *      (2.0/R-GAM-1.0))+1.0)*DRAL-(R-1.0)*DNAL/XN)/XN**2
      RETURN
C
C     OVERCONSOLIDATION ZONE
C
  300 DU=1.0-X*(1.0+YS)
      GAM=-(DU+SQRT((X-YS-1.0)**2+(X*X-1.0)*YS*YS))/(R*(X*X-1.0))
      DFI=2.0*XIO*(GAM-1.0/R)
      DFJ=2.0*XIO*((1.0+YS)/R-X*GAM)/XN
      XKS=DUS*(GAM-1.0/R)*(DU*GAM+2.0*A/XN)/R
      DFJJ=DFJ/SRTJ
      DFAL=6.0*XIO*(DNAL*(TH*GAM/XN-1.0/R+A/(R*TH*GAM)-2.0*A/XN)/
     *      XN**2+DRAL*(1.0/TH-1.0/XN+A/(XN*TH*GAM))/R**2+DAAL*(
     *      1.0/XN-1.0/(TH*GAM*R))/XN)
      RETURN
C
C     TENSION ZONE
C
  400 GAM=(-T+BT-SQRT(BT*BT-RO*TH*TH*T*(T-2.0*BT)))/(1.0+RO*TH*TH)
      DFI=2.0*PSI*XIO*(GAM+T-BT)
      DFJJ=2.0*PSI*XIO*GAM*RO*BS/XIBAR
      DFJ=DFJJ*SRTJ
      XKS=DUS*PSI*PSI*(GAM+T-BT)*(GAM*(BT-T)+T*(2.0*BT-T))
      DYSAL=YS*(DRAL/R+DAAL/A-DNAL/XN)
      DFOPAL=FOP*(DNAL/XN-YS*DYSAL/(1.0+YS*YS))
      DJOAL=XJO*(DAAL/A-DYSAL/YS)+A*(1.0/YS-FOP/XN)*DYSAL
      DBTAL=((T-BT)*DJOAL-(T-2.0*BT)*T*DFOPAL)/(XJO-2.0*T*FOP)
      DROAL=DBTAL/FOP/XJO-RO*(DFOPAL/FOP+DJOAL/XJO)
      DFAL=3.0*PSI*XIO*TH*GAM*(DROAL+2.0*RO*DBTAL/(T+GAM-2.0*BT))
      RETURN
      END
C
C     ********************
C     *                  *
C     *   END OF MODCAL   *
C     *                  *
C     ********************
```

140

END

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